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MODEL STUDIES: THE ULTIMATE BEARING
CAPACITY OF LAYERED COHESIVE SUBSOILS

A THESIS

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MODEL STUDIES: THE ULTIMATE BEARING

CAPACITY OF LAYERED COHESIVE SUBSOILS

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GLOSSARY OF SYMBOLS

The following symbols appear frequently in the thesis; those used, but not listed below, are standard to soil mechanics literature. The numbers following a few of the brief definitions refer to the sections in the thesis where the full definition may be found.

In addition the following are used repeatedly as subscripts: UC, parameter is based on unconfined compression data; vane, parameter is based on vane shear data; avg, parameter given as the average of the vane and UC data.

A_x	= area-cross ratio of shear vane	3.1a
B	= footing width	
b	= one-half the footing width (B/2)	
c	= undrained shear strength	
c_1	= undrained shear strength, top layer	
c_2	= undrained shear strength, bottom layer	
d	= thickness of top layer	
L	= length of footing	
N_c	= bearing capacity factor, homogeneous soil.	1.2
N_m	= modified bearing capacity factor, layered soil . .	1.2
n	= strength ratio, layered soil	1.2
q	= footing pressure	
q_c	= ultimate bearing pressure.	4.1
q_p	= footing pressure at start of perimeter shear . . .	4.1

S_B = settlement ratio =
(settlement/footing width) x 100

ρ = load spread coefficient. . . . . .Appendix, Sec. 2

SUMMARY

This thesis investigates the ultimate bearing capacity of rigid strip footings founded on stratified clays. It involves a laboratory study using model footings on a two-layer clay subsoil of high compressibility. The study has been made using the $\phi = 0$ analysis, considering soil strength under undrained conditions.

Two conditions are examined--stiff layer overlying soft soil and soft layer overlying stiff soil. The results are given in terms of the modified bearing capacity factor, which depends on the thickness of top layer to footing width ratio and also the ratio of the undrained strengths of the component layers. This factor is also found to be greatly influenced by the stress-strain properties of the layers.

For the case of a thin, stiff layer overlying soft soil bearing capacity is developed primarily in the lower layer, preceded by a partial punching failure through the top layer. While for cases of a stiff top layer thickness on the same order as the footing width, bearing capacity is developed in the stiff material. However, the shear planes incline steeply into the soft underlying material, rather than recurving up through the stiff soil as the common theoretical solutions assume.

Soft layer over stiff layer configurations exhibit an extrusion of the top layer material from under the footing. The phenomenon is still present even where the lower layer undergoes plastic distortion.

Recommendations are given concerning the use and limitations of the theoretical methods in light of the testing.

CHAPTER I

INTRODUCTION: A STATEMENT OF THE PROBLEM AND THE HISTORY OF ITS INVESTIGATION

1.1 The Subject of the Present Research

This thesis investigates the ultimate bearing capacity of strip footings founded on two-layer clays. The computation of allowable soil pressure is one of the routine problems of foundation design. The deformations required to cause complete support collapse of the foundation soil, however, are generally so large that the superstructure is likely to be severely distressed long before bearing capacity failure occurs.

On the other hand, in cases where large settlements are tolerated (as in grain elevators, embankments and oil storage tanks) the ultimate bearing capacity may indeed govern the foundation design.

It is significant to note that the cases of failure in the field primarily involved structures of this type, founded on soft clays, and that failure occurred immediately after rapid overloading. With few exceptions, the clay foundations in these cases were not even approximately homogeneous deposits.

For the model tests of the present research, it would have been difficult, if not impossible, to simulate the varied strength-depth soil profiles which occur in natural soil deposits. For most of the field studies noted in the literature, a close approximation is a two-layer or a three-layer soil system. The former was chosen as the test soil configuration primarily because of the nature of theoretical solutions available.

The theories for the ultimate bearing capacity of homogeneous clay foundations have largely been accepted by the profession as sufficiently reliable. These theories have been accompanied by numerous model studies and attempts at field verification. Although it serves no purpose to detail the work in this thesis, it necessarily formed the basis for the study of the layered clay problem. The theoretical solutions have been summarized by Sowers (1963),* while Roberts (1961) has reviewed the related model tests. No such concise summary is extant for the field studies of bearing capacity failure; however, Szechy (1961) presents several of the more important ones.

In contrast to the above, there is general uncertainty over how the layered clay bearing capacity problem should be approached. Many plausible solutions have been advanced, and the methods often lead to widely different bearing capacity predictions for the same problem. Because several of these solutions have not received widespread circulation in the usual soil mechanics literature, they are presented in an appendix to this thesis (see Appendix A).

Finally, since the present research was carried out with model footings, it should be acknowledged that the whole concept of small-scale experiments is regarded with considerable skepticism by many eminent foundation engineers--particularly American engineers. The

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*Works cited are listed alphabetically under References Cited.

objections can be epitomized in one rhetorical question: "Do we build *cigar-box* foundations in field practice?"

This is a simplified way of registering an incisive technical criticism. The bearing capacity "model studies" generally do not attempt to satisfy the requirements of dimensional similitude; i.e., one scales down the footing but not the foundation soil. The difficulties in doing so are manifest. A comprehensive review of testing programs which *did* involve scaling soil through the use of other materials led Roberts (1961) to doubt whether much success will ever be achieved by such techniques.

Since well-controlled full scale field tests are so expensive, there is little comparative data available. The "post-mortems" conducted over unintentional failures in the field obviously lacked good data on soil conditions at the time of loading, or, in some cases, even realization that such data was important.

One can argue in support of model tests with more technical eloquence,* but the central objection, stated above, remains.

It can only be said that small-scale tests are expedient and that the theories which issued from them have not been contradicted by field experiences.

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*In this regard, see: Jumikis, A. R. (1961), Paper 3A/23, *Proceedings,* Fifth International Conference on Soil Mechanics and Foundation Engineering (Paris), pp. 693-698.

1.2 Solutions for the Bearing Capacity of Layered Cohesive Subsoils: Literature Review

The General Equation

The assumption of $\phi = 0$ conditions, that is, a purely cohesive soil, forms the basis for all of the bearing capacity field studies which were reviewed. This assumption has sufficient validity for a saturated clay which is loaded under such conditions that no appreciable consolidation occurs (Skempton, 1948).

For $\phi = 0$ conditions the ultimate bearing capacity, q_c, of a strip footing is independent of footing width and equals a constant (N_c) bearing capacity factor times the undrained shear strength of the soil (c).

An analogous equation is often used to represent the bearing capacity of a two-layer clay

$$q_c = c_1 N_m$$

where

c_1 = undrained shear strength of the top layer, and

N_m = a "modified" bearing capacity factor.

The N_m value is not unique at $\phi = 0$. It varies with the strength ratio of the two component layers as well as the geometry of the foundation system. These independent variables are used throughout this thesis in a notation introduced by Button (1953):

$$\text{strength ratio} = n = \left(\frac{c_2}{c_1} - 1\right)$$

where

c_1, c_2 = undrained shear strength, top and bottom layers, respectively;

and the d/b ratio, where

d = thickness of the top layer, and

b = one-half of the footing width.

1.3 Discussion of the Methods

Perhaps the most significant criticism which is leveled at the existing layered clay bearing capacity solutions is that they were not, in general, accompanied by comprehensive testing programs to determine the limits of their validity. It seems unlikely that one method alone would be valid over the complete ranges of n and d/b. Such methods as Yamaguchi's (1963) have only limited verification, while others (as Taylor, 1948; Button, 1953) were presented with no supporting tests at all. Those methods which were accompanied by complete testing programs are for the specific cases for a thin soft clay layer overlying a very rigid material (Meyerhof and Chaplin, 1953) and a thin stiff clay layer overlying an exceedingly soft clay (Tcheng, 1956). These are the two extremes in the layered soil problem.

Except for the results of three small plate load tests presented by Koizumi (1965), the vast middle range of practical interest was left unstudied* until a recent series of model tests by J. D. Brown (1967). This work continues at the time of the writing; the preliminary results, together with the promise of full detailed treatment to follow soon, were presented by Meyerhof and Brown (1967).

It is the purpose of this section to briefly point out the major assumptions, critical weaknesses and test verification of the methods which are described in Appendix A. The recommendations as to probable range of validity are somewhat speculative in nature, but they represent those of the most substantive references which were found in the literature. The discussion follows the order of the Appendix.

1.3a Skempton's (1951) Empirical Design Recommendations

Skempton's (1951) empirical design recommendations assume the shear zones extend only to a depth of about two-thirds times the footing width, irrespective of the nature of the non-homogeneity, provided that the strength variation is not a drastic one (> 50 per cent) in that depth. Model tests on layered clays, including those of the present research, indicate that the shear zones extend much deeper--at least for some soil system geometries.

Skempton drew upon model tests by Golder and Meigh (1950)** plus available field data to establish his rule. In a later field study

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*Both Tcheng (1957) and Brown (1967) reference a paper by L. Suklie (1954), presented in *Proceedings*, Yugoslav Society of Soil Mechanics and Foundation Engineering, No. 3, in which the results of two model footing tests involving the soft over stiff layer configuration were discussed. This paper was not read for the present study.

**The latter paper was not read; Skempton (1951) references it thus: Meigh, A. C. (1950), "Model Footing Tests on Clay," M. Sc. (Eng.) Thesis, Imperial College, University of London.

Bjerrum and Overland (1957) analyzed the Fredrikstad tank failure by direct application of Skempton's approximation and found a discrepancy of 72 per cent between actual and computed bearing capacity. However, the investigators suggested that the tank underwent a "localized" (edge) bearing capacity failure. With this assumption, Skempton's method yielded an overestimate of only 5 per cent. It is suspect, however, since the true area of the tank segment which participated in the failure was unknown; several reasonable guesses were tried and the one which gave the minimum ultimate bearing capacity by Skempton's method was used in recording the calculated factor of safety of 1.05.

Similarly using Skempton's method Eden and Bozozuk (1963) analyzed the failure of a silo founded on soft varved clay. For the estimated load range of 550 ± 50 tons, they computed safety factors ranging from 0.97 to 0.80.

The field studies described were both for a stiff layer over soft layer configuration; Skempton's approximation is apparently a reasonably good estimate for this condition. The model tests of Brown (1967) indicate that the method has applicability, in the two-layer case, where $-0.3 \leq n \leq 0$ or $d/b \leq 0.2$. In fact, by taking the average strength for a depth of 0.8 (instead of 0.66) times the footing width, Skempton's method and Brown's (1967) limit plasticity solution (presented below) yield identical results in this range.

1.3b The Load Spread Techniques

The load spread techniques, such as those of Taylor (1948), all neglect the strength contribution of the stiff top layer. They therefore must assume that the soft underlying soil can be strained to failure without shearing the top layer. The methods are, as stated by Taylor, "rough indications" of bearing capacity which are considered as conservative.

The load spread proposals were all based on intuition and were not accompanied by laboratory model tests. Nevertheless, the methods have been frequently used in bearing capacity field studies. Nixon (1949) used a $22\frac{1}{2}°$ uniform load spread in analyzing the Shellhaven tank failure and found close agreement with Terzaghi's (1943) bearing capacity factor $N_c = 7.4$ (for a round footing on homogeneous $\phi = 0$ material). This involved a load spread approximation for in-situ conditions of $d/b = 0.055$ and $n = -0.7$.

In a discussion of Nixon's analysis Saurin (1949) criticized the load spread concept. The latter's analysis of the Grangemouth tank failure ($d/b = 0.021$; $n = -0.75$) considered the full load of the tank and stiff top layer on the surface of the soft material with no spread of load. The analysis checked with Terzaghi's (1943) "local shear" bearing capacity solution. Saurin made several approximate strength corrections and his choice of what constituted the ultimate load has made the analysis subject to doubt (Raymond, 1967).

It might be concluded that the load spread technique works reasonably well where d/b is small ($\lesssim 0.05$) or where the soil system has an $n \lesssim -0.7$. The former conclusion can be drawn from field studies, and the latter from the fact that if the soil system is a near-homogeneous one (n approaching zero) then the strain required to mobilize the bearing capacity of the lower layer will have caused significant plastic distortion of the top layer, destroying any alleged load spread capability. The solution may not always be conservative therefore. It is significant to note that the mechanism of load spread was not found operative in model tests (Tcheng, 1957) which later followed, leading to the conclusion that the solution involves a fortuitous counterbalancing of errors.

Considering the approximate nature of the method, Taylor (1948) suggested that the 30° Boston Code spread is as good as any distribution. However, since the technique cannot always be relied upon to give safe-side predictions, there seems to be no reason to ignore the slightly more conservative Nixon 22 $\frac{1}{2}$° spread which has more theoretical validity.*

1.3c Yamaguchi's (1964) Modified Load Spread Technique

Yamaguchi's (1964) modified load spread technique accounts for the strength of the top stiff layer by introducing the term d/3b into a conventional load spread equation (see Equation A.7). His choice of failure plane through the top layer, it would appear, was governed by ease of analysis; Yamaguchi intended it to be a simple and "practical" formula.

Since no tests were conducted on layered clays, its range of validity for that case was not indicated by Yamaguchi. Later model tests, including those of the present study, indicate that the shear

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*Nixon (1949) calls the 22 $\frac{1}{2}$° uniform load spread "an approximate Boussinesq distribution," citing previous work of Glossop and Golder in Road Paper No. 15, Inst. Civil Eng., London.

planes do not propagate back through the top layer. This is especially true in the range of n values where the conventional load spread is considered to be most conservative.

An additional shortcoming of both Yamaguchi's method and the conventional load spread techniques is that they offer no solution for the case of soft soil overlying stiff soil.

1.3d Button's (1953) Method

Button's (1953) Method assumes that the non-homogeneity of the subsoil will have no effect on the shape of the failure zone. He retained the slip circle of the homogeneous $\phi = 0$ analysis, ignoring the fact that in a layered soil slip surfaces may be composite or discontinuous, with actual failure occurring along that potential slip surface which has a minimum bearing capacity.

It is important to note that Button's solution satisfies the plasticity theory requirements for an upper bound solution (see Drucker and Prager, 1952), i.e., it constitutes a kinematically admissible flow field at impending failure. The results by Button's method can therefore be expected to predict too high a value of bearing capacity over the complete range of n and d/b values. The objections on this point are crystalized in the comment of Schofield and Wroth (1968):

> . . . the slip circle calculations would be all right for demolition experts who wanted to be sure to order enough load to cause failure, but civil engineers who want to be sure of not overloading the soil ought first to think of stress distributions. (Lower bound solutions.)

Button performed no companion tests to verify his solution. It is difficult to assess the magnitude of the overestimation involved in using his method. The preliminary results of Meyerhof and Brown (1967) provide the best available indications (see Figure 1).

In one important respect, Button's solution is superior to methods which assume the seemingly more accurate Prandtl geometry (as Tcheng's (1956) Prandtl solution). Button searches for the worst possible condition; i.e., the strength ratios do have an effect of the depth of the assumed shear zone (albeit, none on the shape of the zone).

1.3e Tcheng's (1956) Prandtl Solution

Tcheng's (1956) Prandtl Solution is another solution of the upper-bound type. Its assumption of failure geometry, accurate for the homogeneous case, cannot be expected to apply for a great range of strength ratios. Tcheng, recognizing this, later modified his solution to compare more closely with test observations.

1.3f Brown's (1967) Limit Plasticity Solution

Brown's (1967) Limit Plasticity Solution is a refinement of the above approach. As all other solutions, it assumes complete mobilization of shear strength simultaneously in both layers, i.e., strain incompatibility is ignored. The limitations on the use of plasticity theory is one of the current topics in soil mechanics research. It appears that very little is known about displacements (especially in clay) at failure. Nevertheless Brown's tests noted good agreement with the average value (Equation A.25) where $n \geq -0.3$ or $d/b \leq 0.2$. This could be taken as circumstantial evidence of the applicability of the plasticity solution.

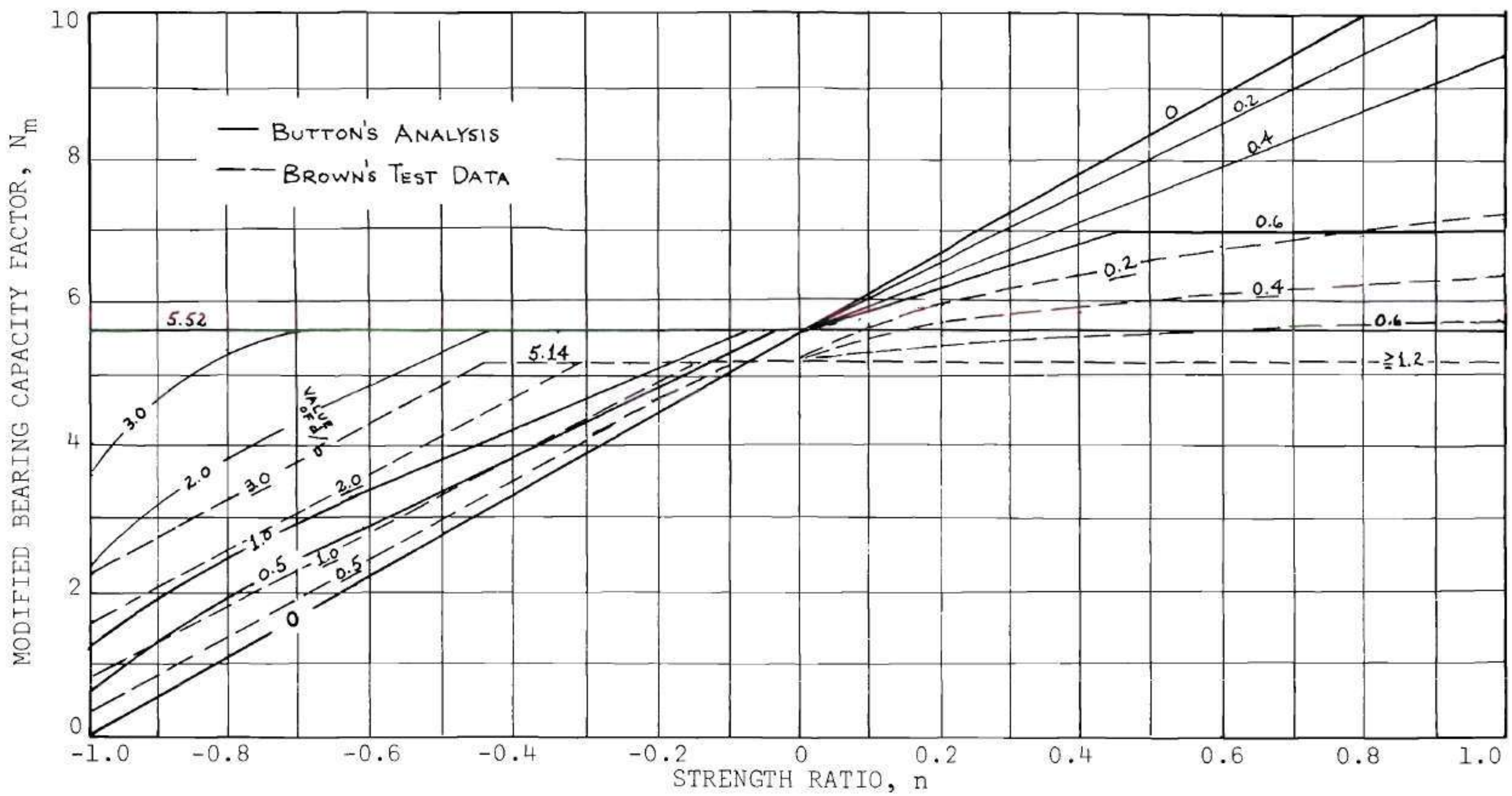

Figure 1.1 Experimental Values of N_m Against Those of Button's (1953) Method (after Meyerhof and Brown, 1967)

1.3g The Meyerhof and Chaplin (1953) Theory

The Meyerhof and Chaplin (1953) theory is a theoretically correct solution, verified by model tests. It appears from tests by Brown (1967) that, with certain modifications, this theory describes the failure geometry obtained over much of the +n range of strength ratios. However, Brown's modification is not based on the $\phi = 0$ analysis, but rather on the true angle of internal friction (for his soil, $\phi_e = 22^o$) so that it cannot be generally applied.

1.3h Tcheng's Prandtl Punching Solution (1956)

Tcheng's Prandtl punching solution (1956) assumes straight vertical punching through the top layer. He tested soil systems in the low -n range ($n \approx -0.96$) and gave no indication as to range of applicability. Koizumi (1965) indicated that vertical punching probably occurs in a two-layer clay system where $d/b < 1.16$ and $n < -0.84$.

1.3i The Results of Brown's (1967) Footing Tests

The results of Brown's (1967) footing tests indicated that a "fairly good representation" of his data was obtained by Equation (A.29) for the range $n > -0.3$ or $d/b > 0.25$, while the plasticity solution (Equation A.25) applies outside this range for the stiff over soft configuration.

It should be noted that the *aborted shear punching solution* given by Equation (A.29) is not to be considered applicable to all soil systems of stiff over soft layer. The degree of punching attained will depend on the stress-strain properties, as well as the tensile strength of the top layer, i.e., a diagonal tension failure through the top layer is possible for a very lean clay.

A summary of the methods is noted in Table 1.

Table 1. Summary of Existing Methods of Finding the Bearing Capacity of a Strip Footing on Layered Clay

Method	Equation for N_m, Modified Bearing Capacity Factor*	Use
Skempton's (1951)	Bearing capacity directly determined by average c in homogeneous q_c equation	$d/b \leq 0.2$ or $-0.3 \leq n \leq 0$, with $N_m = N_c = 5.14$
Taylor (1948), Nixon (1949) load spread	$[(n+1)(5.14 + \rho(d/b))]$	$d/b < 0.05$; $n < -0.7$ use $\rho = 2.13$ (Nixon, 1949)
Yamaguchi (1963) modified load spread	$[(n+1)(5.14(1 + \frac{d}{2b})) + d/3b]$	Solution nearly identical to Nixon (1949) method for range of $d/b < 0.05$
Button (1953) slip circle analysis	Graphically determined	Values too high throughout range; best available solution soft/stiff configuration where $b/d \geq 2$
Tcheng (1956) Prandtl solution	$5.14(n+1) - 2(d/b)^2 n$, $d/b \leq 1$ or, $4\beta n + 5.14$, $d/b \geq 1$	Approximately same range as Brown's limit plasticity solution (see below)
Tcheng (1956) Prandtl-punching solution	$d/b + 5.14\ (n+1)$	Accurate for limiting case $n \to -1$ perhaps to $n = -0.85$ for $d/b < 1.2$
Brown (1967) limit plasticity solution	$3.25\ d/b + (n+1)(5.14-3.25\ d/b)$	$d/b \leq 0.2$ or $-0.3 \leq n \leq 0$
Brown (1967) aborted shear punching solution	$0.75\ (d/b) + 5.14\ (n+1)$	$d/b \geq 0.2$ or $n \leq -0.3$
Meyerhof and Chaplin (1953) theory	$\pi + 1 + b/d$	$d/b \leq \frac{1}{2}$ probably true for complete $+n$ range

*Where ultimate bearing capacity equals N_m times undrained strength of top layer.

CHAPTER II

TEST APPARATUS AND PROCEDURE

2.1 Test Materials

2.1a Footings

Two strip footings were used in the testing program. They were composite constructions using standard aluminum shapes. Both had a length-width ratio (L/B) of six. The theoretical solutions are based on $L/B = \infty$. Since in a layered soil system, bearing capacity is a function of layer thickness, the validity of "shape factors" as proposed by Terzaghi (1943) for the homogeneous case, is highly problematic. The assumption: $L/B = 6 = \infty$ therefore becomes requisite.

To assess the size of the error introduced by the above assumption, a semiempirical evaluation proposed by Tschebotarioff (1951) was applied. It was thus estimated that (for homogeneous clay, under $\phi = 0$ conditions) the test footings developed about $7\frac{1}{2}$ per cent greater bearing capacity than an infinite strip. While this cannot be quantitatively applied to the layered clay case, it shows that the assumption of an infinite strip is not an unreasonable one.

The bases of both footings were epoxy painted and roughened with approximately one grain thickness of standard Ottawa sand (Figure 2-1). The Prandtl (1920) theory, which sired most of the analyses of Appendix A, assumes a perfectly smooth footing base. However, Liam Finn (1967) shows theoretical evidence that the Prandtl figure is associated with a

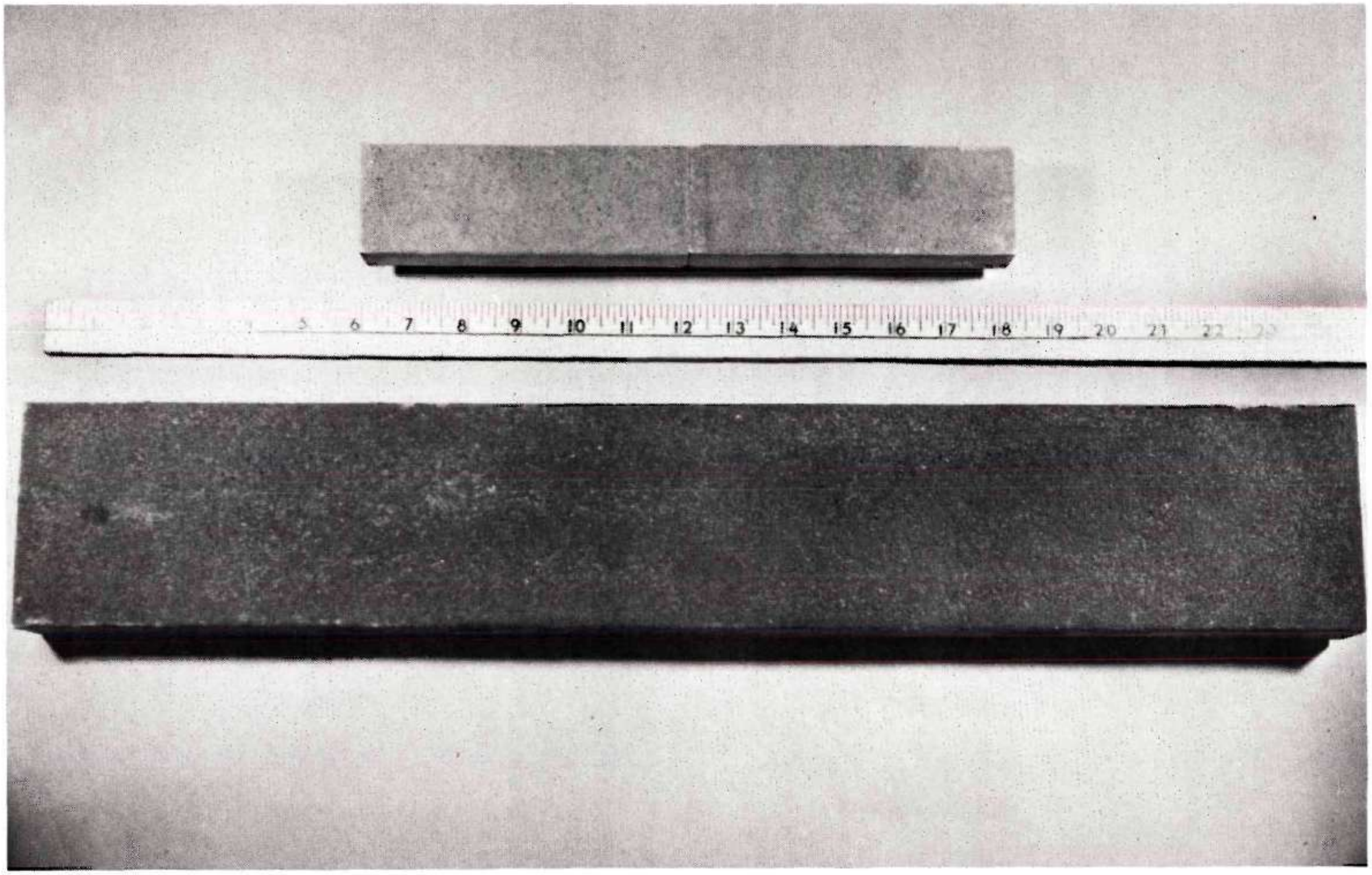

Figure 2-1. Rough Bases of the Test Footings

sufficiently rough footing. In any event a rough base is generally encountered in actual foundation practice.

The smaller (2" x 12") test footing was of special split design (Figure 2-3) and will be discussed further in the ensuing text. The larger (4" x 24") test footing was a single piece design. Both footings were considered as being completely rigid under the relatively small contact stresses engaged in the various tests.

2.1b Test Boxes

Since there is little information in the literature on the shape and extent of failure surfaces in layered soils, and indeed none for soils of the strengths used, proper test box dimensions were arrived at through consideration of the failure geometry for the homogeneous case.

Meyerhof (1950) suggests that the test box be considerably larger than the extent of the potential failure surface because movement can be detected at distances two to three times the limit of the failure surface. However, other studies (Roberts, 1961), in which it was considered important only to have the box larger than the failure surface, seem to give reasonable results.

The initial (2LS) series of footing tests was carried out (using the 2" footing) in two steel containers, each 21" x 17" x 7 $\frac{1}{4}$" deep. For cases involving a stiff, thick top layer overlying soft soil, later research showed that the 7 $\frac{1}{4}$" depth was probably insufficient.

Although only three tests (series 2LL) using the larger (4" wide) footing, the limited time available for testing dictated that two test boxes be used. Both were 24" x 48" x 18" deep. One box was constructed of 3/8" steel plates bolted securely with 2" steel angles.

The other box was fabricated from 3/4" plywood sheets, reinforced with 2" x 4" pine beams to keep the sides from flexing unduly. The wooden box was waterproofed with two coats of epoxy paint.

A series (2LB) of tests using the 2" footing with its split capability was also performed in a 24" x 24" x 16" deep plywood box of 3/4" wall thickness, again latterally reinforced with 2" x 4" pine boards. This box was designed to be split in half to observe slip surfaces, if any, under the failed foundation.

2.1c Test Soil

The terms "stiff" and "soft" will be used in this thesis in a highly restricted, relative sense. The so-called "stiff" soil is itself of very low strength.

A great deal of soil was required in this testing program. It was desired to maintain as high a level of control over the composition of the soil layer components as possible. The native soils of the Atlanta region are notorious for their heterogeneous nature. Therefore, it was decided to use commercially processed soils. Those available in quantity were pulverized Wyoming bentonite and Georgia kaolinite. Index properties of these materials are listed in the table following.

Table of Index Properties

	Stiff Soil	Soft Soil
Classification (Unified System)	CH	CH
Specific Gravity	2.67	2.65
Liquid Limit	548	456
Plastic Limit	58	65
Plasticity Index	490	391
Sensitivity, S_t, by Vane (Using four-day setting time)	2.0	1.7

A strength difference could have been realized by simply using either homogeneous bentonite or kaolinite through varying the water content. This would have required some method for preventing moisture migration across the interface, i.e., an impervious membrane separating the two layers--accentuating an already obvious plane of weakness in the soil system.

It was rather decided to achieve the strength difference by variation of the mineral composition of the layers. The soft soil was composed of three parts bentonite to one part kaolinite (dry weight proportion) at nominal 325 per cent water content; the stiffer soil was composed of homogeneous bentonite also at a water content of 325 per cent. (A detailed discussion of strength and other properties of the test soils is presented in a following section.)

This composition satisfied strength ratio and placement requirements, while retaining what advantages there are in using bentonite:

> The soil could be placed in lifts which later "heal" into a singular mass with no evidence of compaction plane weakness. It also allows placement at a very workable consistency with strength achieved in later thixotropic regain.

The use of this soil system caused at least two very serious problems. First, the great footing penetrations developed in the course of a test resulted in much intermixing of component layer materials. The amount of soil which could be salvaged for reuse was therefore quite limited. Second, because the soil system manifested large strains before failure, results are somewhat compromised. It is tacitly assumed in the theories that the footing penetration at failure is negligibly small with respect to the footing width. While it might be argued that no soil exhibits such perfect plasticity, the problem remains and is aggravated by the fact that, in layered soil bearing capacity theory, the yield pressure, q_c, is a function of (d/b). In this case, the layer geometry at $q = 0$ is significantly different from that at $q = q_c$. Some attempts must be made to consider this in the interpretation of results.

2.1d Mixing and Placement of Soil

The soil was mixed in batches varying between (approximately) 40 and 110 pounds, depending on the layer thicknesses required for a particular test. All weighings were done on a Toledo scale, reading to 0.1 pounds. The soil was blended, from the powdered state, in a large Readco mixer, using an open-spade mixing bit. The nominal mixing time was 30 minutes per 50 pound batch; however, none of the mixing tools available provided the desired kneading action. Rather, the mixer had to be repeatedly stopped, as the clay tended to cohere in large wads and to veneer the sides of the mixing bowl. This lack of uniformity of mixing, in itself, was a major factor in determining what strength ratios were actually obtained.

For the 2LS and 2LL series of tests, the placement technique was similar. The soil for the bottom layer was placed in approximately one inch lift thicknesses. It was molded into place with the fingers and then troweled or floated carefully to remove air voids. The surface was then randomly lacerated with a wire brush and then the next lift placed.

At the outset of the testing program, some attempts were made to formulate a standard compaction technique. All were futile. The procedure used is admittedly crude; certainly, exact reproducibility of strength ratios for different tests was coincidental. It should be emphasized that strength, water content and density data were obtained in each test on the soil in the *in-situ* condition.

When the level of the two layer interface was reached, a set of graduated aluminum guides was set in place, and the final surface of the bottom layer was repeatedly screeded, troweled and re-screeded using a steel straight edge. When the desired planeness was achieved, this surface was then very lightly striated with a straw brush in the probable failure zone, parallel to the long axis of the footing. This provided some lugs to aid shear transfer over the layer interface.

Examination of the soil system after the tests indicated the maximum variation in top layer thickness, d, was typically ± 1/16" or about 3 per cent of the small footing width. It was therefore desirable to use the larger footing for cases where top layer thickness, d, was to be small.

Mixing and placement time for each 2" x 12" footing test soil was approximately four hours, while for the larger boxes of the 4" footing tests the time was eight to ten hours. Since each of the large tests required nearly one-half ton of soil, which was mixed and placed each time without assistance, close control over evaporation losses was not possible.

In order to provide a visual record of failure deformations of the subsoil, a special banding technique was employed. It is well to make a clear distinction in terminology here. The banded subsoil is still a two-layer system. The addition of 0.04 per cent lampblack (by total weight of soil) caused no significant difference in strength (as measured by the largest vane cross) from the ordinary light-colored soil.

The soil system for the banded tests was formed by the same techniques as described above except that each layer was formed in nominal $\frac{1}{4}$" alternate light and dark bands. A total banded, layered soil thickness of at least three times the footing width was maintained. Preparation time was about 17 to 20 hours per test.

2.2 Test Procedure

2.2a 2LS Series (2" x 12" Footing); 2LL Series (4" x 24" Footing)

These tests involved obtaining bearing capacity data without observation of the subsurface deformations. The 2LS series was performed as follows.

The loads were applied by dead weight in either one or two pound increments using a piston traveling through a Thompson bushing mounted on a frame spanning the test box. The piston was merely rested in a saddle at the top of the footing.

Model footing tests are often conducted using a piston rod threaded into the footing to provide rotational restraint; such a condition satisfies Prandtl's (1920) theory, which was for a *guided,* lubricated metal punch, but is a highly questionable one for a structural foundation. Deformations were measured using a type 25 Starrett dial gage reading to .001" mounted on the frame.

Care was taken to avoid eccentricity of loading, since piston friction was not assessed, but it could not be eliminated, and in some cases, rotation of the footing was pronounced. The 2LL series involved loading the footing by means of a four square inch Bellofram, the ram of which was threaded into the top of the load piston, traveling through a Thompson ball bushing. A five inch diameter proving ring (7.7 lbs/div) was mounted on the footing and the piston was brought down to meet a saddle in the ring's top mounting plate (see Figure 2-2). The loading increment was varied depending on the anticipated ultimate, but usually was set at 15 psf. Three Starrett gages were mounted over the long axis of the footing to obtain average settlement. In all cases loading was carried well beyond the theoretical bearing capacity to supply a complete pressure-settlement curve.

In the 2LB split box series, the loading was as in the 2LS tests, but as rapid settlement began, the load was taken off. The load tube of the split footing (see Figure 2-3) was removed, leaving the split

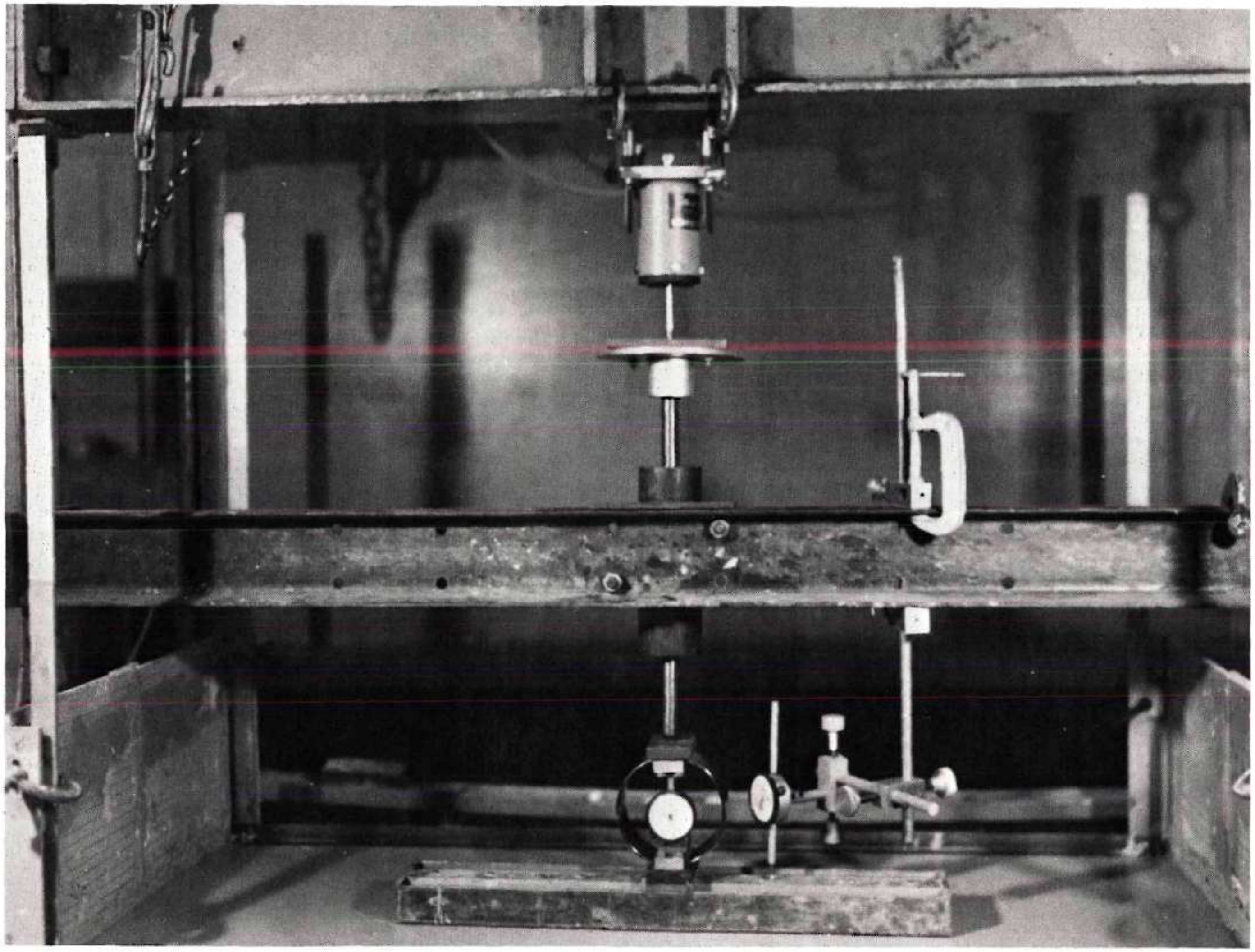

Figure 2-2. Footing Test Apparatus for the Large (2LL) Model Series

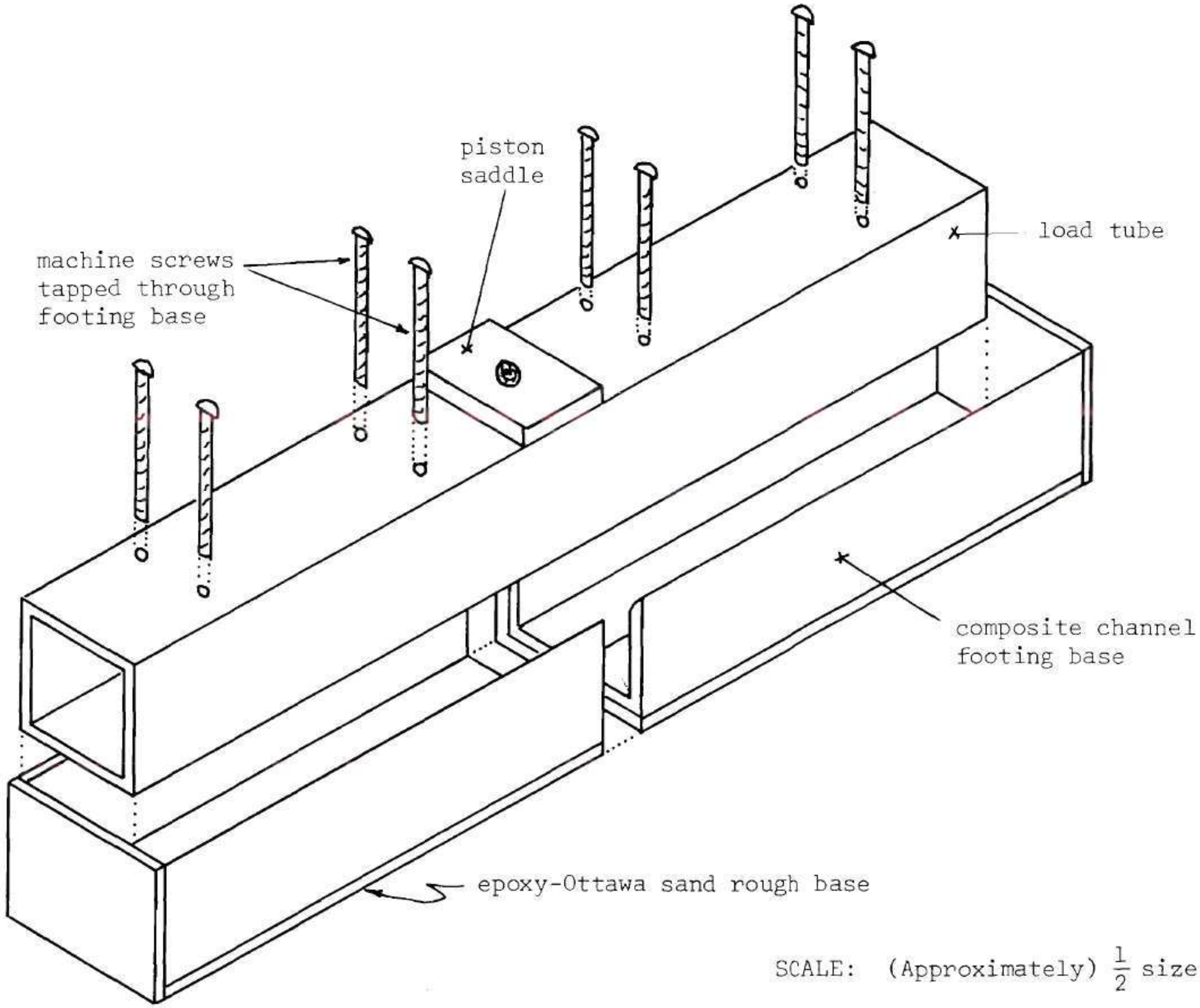

Figure 2-3. 2" x 12" Split Strip Footing, Series 2LS and 2LB

footing base in place. The restraining clamps of the split box were removed and a piano wire saw passed through the box. It was then opened for examination of the failure. Although this technique in many respects is inferior to the glass-sided box method often employed in such studies (Tcheng, 1957; Brown, 1967), it eliminates the problem of parasite friction between glass and soil, and recognizes the fact that plane strain conditions may not be a good approximation for an L/B = 6 footing throughout the range of n and d/b tested.

CHAPTER III

STRENGTH TESTING OF THE SOIL SYSTEM COMPONENTS

3.1 General

As evidenced in such details as the roughened bases of the test footings, their condition of rotational freedom, and stress (as opposed to strain) controlled loading, it was desired to simulate typical conditions encountered in the prototype. Efforts were therefore made to interpret the test results in a manner which could be related to common field practice.

In routine foundation work undrained shear strength is estimated by the standard penetration test, or, more reliably, by vane shear tests or unconfined compression tests. The latter two methods had application to the present laboratory research. It is not intended to imply that either the vane or unconfined compression tests' results yield a "true shear strength," but rather that they are representative of the soil strength under the actual (undrained) conditions of loading.

3.1a Vane Shear Tests: Apparatus

The vane shear strength data was obtained with a laboratory vane test machine manufactured by Leonard Farnell Co. (see Figure 3-1). This device is equipped to measure applied torque in terms of torsional spring deflections. The three vanes used were all of the four-bladed cross type (see Figure 3-2), and each had a height-diameter (h/D) ratio of one. They were of nominal size $\frac{1}{2}$", 1", and 1 $\frac{1}{2}$". The smallest vane

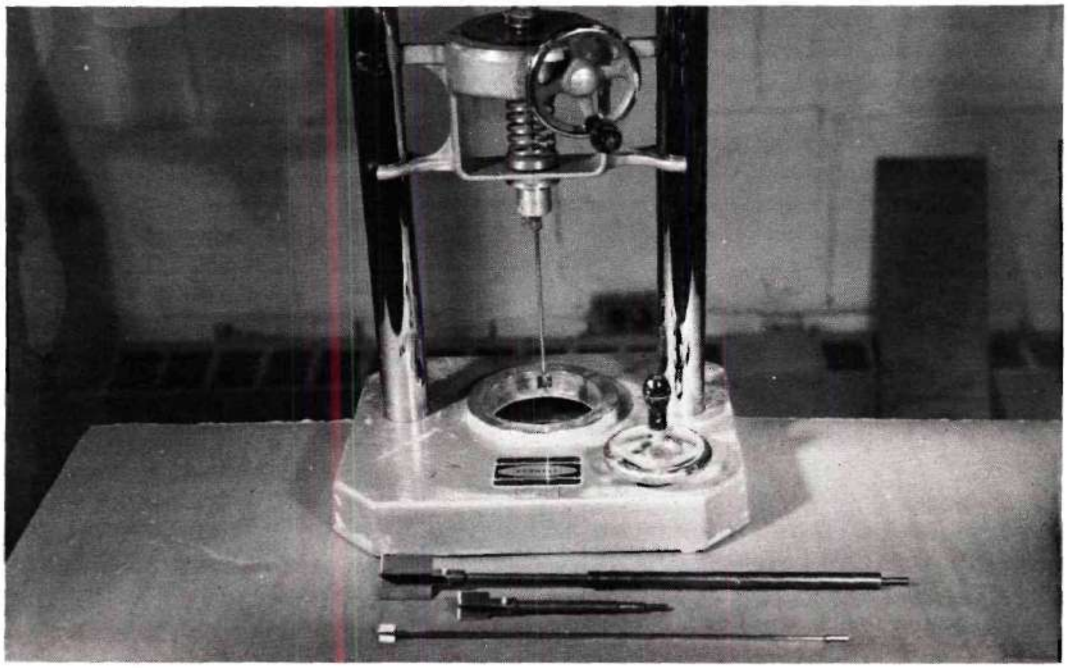

Figure 3-1. Farnell Vane Test Machine with Vane Crosses

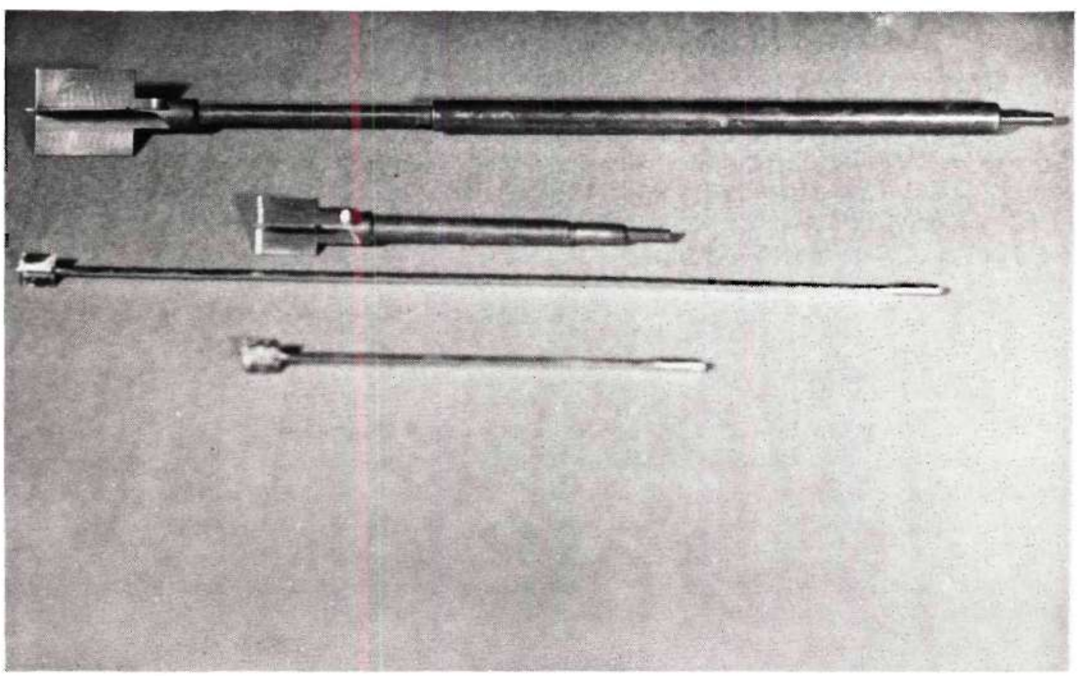

Figure 3-2. Vane Crosses Used in Shear Testing

cross is the standard one provided by Farnell.

The area-cross ratio A_x is defined herein as the cross-sectional area of the vane cross plus stem in percentage of the cross-sectional area of the circumscribed cylinder. Flaate (1965) suggests that A_x be no greater than 15 per cent. The Farnell standard $\frac{1}{2}''$ vane had an A_x of 24 per cent, however, the test soils are in the low sensitivity range.

Because of the extreme softness of the test soils, the standard vane did not engage much resisting torque. To obtain more reliable shear strength values, the 1" and $1\frac{1}{2}''$ vane crosses (A_x = 14 per cent and 10 per cent, respectively) were machined from steel bar stock and designed with an extension sleeve to fit the Farnell device.*

3.2b Vane Tests: Procedure

At the conclusion of each footing test, vane shear tests for both layers were performed both inside and well outside the shear zones. Layer thicknesses governed which vane cross was used, but in later tests the larger ones were used wherever possible.

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*These vanes were not "calibrated" in the strict sense of the word since the torsional spring constants provided by Farnell were used directly. The technique employed was then based on geometrical relations using: $T = c\pi(Dh/2 + D^3/6)$ where T = applied torque at failure; c = undrained shear strength of the soil. This equation assumes linear distribution of shear stresses along the ends of the circumscribed cylinder. In theory (Flaate, 1965) as well as in practice (Osterberg, 1957) this has been shown to be a sufficiently accurate assumption.

The calculated ratio of larger vane torque to the standard (one-half inch) vane torque, T_1/T_S is proportional to the deflection angle ratio (α_1/α_s) for a particular torsion spring. Since α vs. c curves were supplied, the α_1 vs. c plots required could be drawn.

The three vane crosses were tested in soft soils of various consistencies, and in all cases the several vanes gave overlapping strength values, with no consistent, discernible pattern of deviation.

The length of the extension sleeve limited vane tests to a maximum depth of about six inches; however, this was considered sufficient for the range of layer thickness used in the testing program. The rate of strain for vane tests has been generally standardized at 0.1 degrees per second (Osterberg, 1957). In this research it was considered important to maintain approximately the same strain rate in each test, but no strict time controls were set.

3.3 Unconfined Compression Testing Apparatus and Procedure

The extreme softness of both component layer materials made it impossible to carve specimens in a trimming lathe, or indeed to handle a specimen at all with the fingers. Therefore push tube samples were taken using an aluminum thin-walled tube of nominal three inch length by 1.4 inch diameter. The tube had an area ratio* of approximately 27 per cent with a cutting edge feathered at a 10° angle.

Samples were taken, whenever possible, inside the test box after the footing test well outside the shear zones. In cases where the top layer thickness was insufficient to obtain an in-situ tube sample, test soil was placed in a metal trough 24" x 8" x 6" deep, compacted and cured in the same manner as the test box soil.

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*Area Ratio $= \dfrac{D_\omega^2 - D_e^2}{D_e^2} = C_a$

D_ω = tube outside diameter

D_e = tube inside diameter

The sampler tube was pushed by hand in one continuous motion, using a steel push block. Plumbness was maintained by means of a fish-eye level mounted on the block. The specimen was jacked out of the tube (in the same direction as soil entry) directly into the final test position. The sample was capped with a porous stone and load with a modified blunt-point penetrometer. This device was made from a Federal dial gage (reading to 0.001") by removing the spindle spring and threading a small aluminum load platform into the spindle. Loading was done with a series of laboratory metric weights.

3.4 Undrained Strength: Vane Shear vs. Unconfined Compression

3.4a General

The correlation between undrained strength as determined by the vane shear test and that taken as one-half the unconfined compression strength (vane-UC ratio) has been the subject of much research, and in many ways is a subject well beyond the scope of this thesis.

In this testing program the vane-UC ratio established considerable scatter (see Figure 3-3). The methods by which strength data were obtained in this testing program admit of little procedural elegance; however, far more sophisticated testing programs (e.g., Marsal, 1957; Miller, 1957; Flaate, 1965) have not been able to establish definite vane-UC relations, much less that this ratio is unity.

A problem immediately arises as to which test should be used to determine the undrained shear strength, that is, which test most closely simulates the conditions of shear in the footing test. In this regard

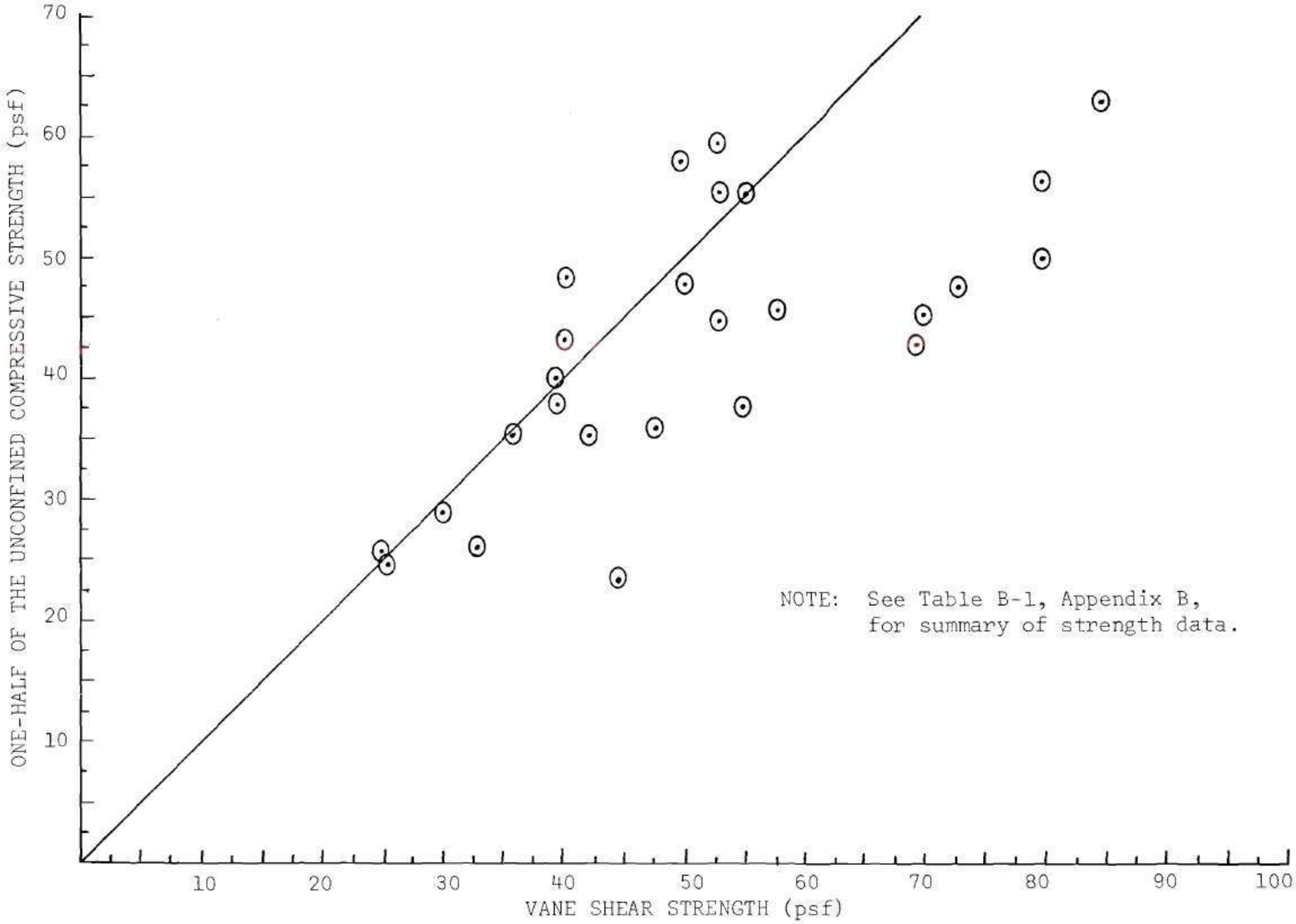


Figure 3-3. Test Results: Vane Shear vs. One-Half the Unconfined Compressive Strength

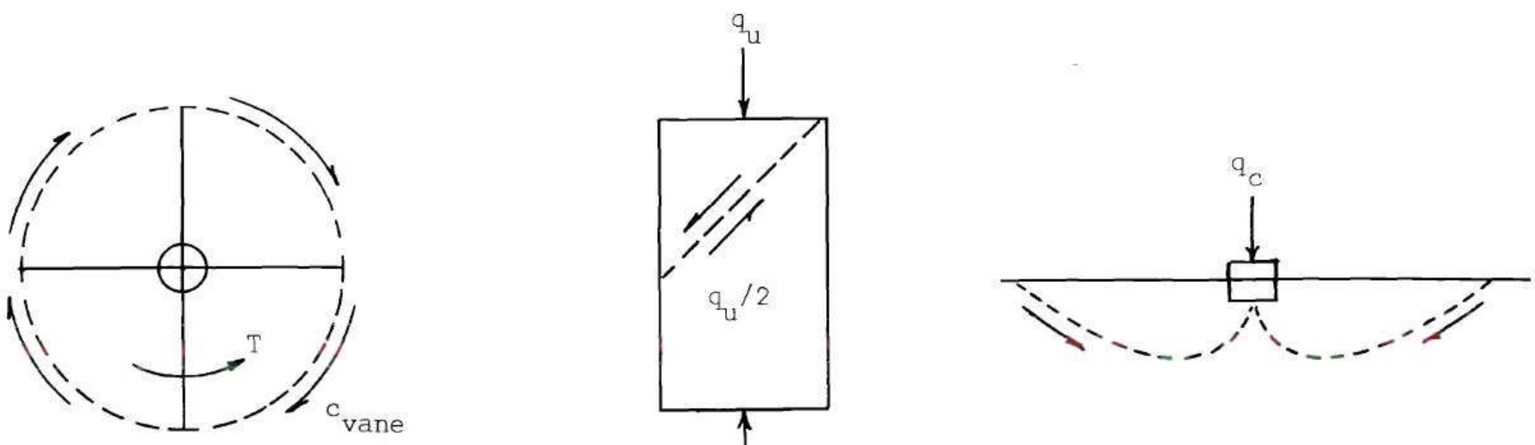
it is highly pertinent to know *why* the vane and unconfined compression tests yield different strengths in order to assess their reliability in bearing capacity computation.

3.4b The Modes of Failure: Vane, Unconfined Compression, and Model Footing Tests

As shown in Figure 3-4, the three tests all result in different shear plane formation; Lo and Milligan (1967) suggest that, for the $\phi = 0$ analysis, the undrained strength at any point in a soil mass should be chosen according to the orientation of the failure surface assumed. At present there is no apparatus capable of measuring field shear strength in different directions; the cost of block samples to determine strength anisotropy is prohibitive in ordinary foundation work.

It can be expected that the placement procedure of the laboratory test soils caused an induced anisotropy, in some respects similar to the natural counterpart in the field. However it was not considered in the testing.

In the recent literature (Srinivasan and Siva Reddy, 1967; Raymond, 1967) anisotropic strength considerations have been theoretically treated in modifications of the slip circle (Button, 1953) layered soil solution. Nevertheless they have been criticized (Meyerhof and Brown, 1967; Ramanathan, 1967) as perhaps unwarranted refinements to a fundamentally approximate solution. The discussion will therefore concentrate on the purely mechanical reasons for the variable vane-UC ratio.



Strength on vertical planes, predetermined;

slip zone forced to be unnaturally thin;

little disturbance.

(a) Vane Shear

Strength on weakest oblique (≈ 45°) plane;

thickness of failure zone ≈ footing test;

sampling disturbance.

(b) Unconfined Compression

Strength composite log spiral and straight lines; possibility of vertical punching in stiff over soft two-layer configuration.

(c) Footing Test

Figure 3-4. Comparison of Failure Modes: Vane Shear, Unconfined Compression, Footing Test, ϕ = 0 Conditions

3.4c Vane-UC: Influence of Soil Macrostructure

Any non-uniformity of structure will be reflected in the vane-UC ratio. This ratio would be greater than one since the vane shears soil along a fixed vertical* surface, whereas the q_u test allows the specimen to shear along the weakest surface.

Goughnour and Sallberg (1964) found that for "highly plastic" soils (PI - 30 to 50) the average strength measured by laboratory vane test exceeded that measured by unconfined compression by 40 per cent. They state that it is likely, as the PI of the soil increases, the uniformity of a molded test specimen decreases.

For the soils of the present tests PI values were about ten times that of Goughnour and Sallberg's tests. Furthermore small voids were randomly distributed through the test soil mass, so that "macrostructure" would seem to be an important influence in setting vane-UC ratios.

3.4d Vane-UC Ratio: Influence of Sampling Disturbance

The vane tests are performed in-situ while q_u tests are generally performed on tube samples. The latter is therefore subject to considerably more disturbance. This disturbance has been regarded as perhaps the major factor in obtaining vane-UC ratios greater than one in both soil of high sensitivity (Gray, 1955) and medium sensitivity (Marsal, 1957).

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*The end areas of the vane cross are small and shear strength is only partially mobilized there at maximum resistance torque; hence, the strength on a vertical surface is essentially measured.

Although the soils of this testing program have low sensitivities (S_t = 2, by vane), they are also exceedingly soft. Fenske (1957) reported results of unconfined compression tests and laboratory vane tests on low sensitivity (S_t = 2.5 by vane) clays of the Louisiana Gulf coast. He found shear strengths by the two tests to be similar in magnitude. The only major exceptions to this similarity were soils that were too soft in consistency (c_{vane} = 125 psf) to perform unconfined compression tests without appreciable remolding. In these cases, shear strengths from the miniature vane tests were appreciably higher than those from unconfined compression.

There are other considerations, such as differences in rates of shear,* progressive failure effect,** and pore water pressure changes** which are also hypothesized in setting the vane-UC ratio, but the two factors discussed above are not subject to doubt.

3.4e Use of Vane-UC Data in the Bearing Capacity Computations

The computations based on vane shear and those on unconfined compression will be separately presented. From the above discussion, it would appear that a vane-UC ratio close to unity can be achieved where sampling disturbance is slight, compaction stresses are largely dissipated and structural irregularities are at a minimum.

In certain cases in the testing program the footing essentially punches vertically into a stiff top layer. It would seem that the vane

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*Housel, W. (1957), Discussion in Symposium on Vane Shear Testing of Soil, ASTM, STP 193, pp. 63-65.

**Burmister, D. (1957), *ibid.*, pp. 65-68. The conclusions of J. O. Osterberg, pp. 68-70, make some critical comment on the above discussions.

shear test would give better results in such a case. Therefore, in instances where the vane-UC ratio is not greatly in excess of unity, a combined version of vane C_1 and unconfined compression C_2 is also used in some added computation as a comparison.

CHAPTER IV

TEST RESULTS AND DISCUSSION

4.1 General

It has long been recognized (Terzaghi, 1943) that soft, compressible clays, such as those used in this testing program, are very poor approximations to the purely plastic models of the theories. In the case of soft soils, bearing capacity is more dependent on settlement; the stress-strain characteristics are such that the soil cannot transfer stresses outward latterally, and so-called "local shear" failure occurs.

In the split box test series, it was observed that in no case did the shear planes extend latterally much beyond the footing width. The pressure-settlement curves presented in this chapter generally exhibit a marked zone of transition before unlimited settlement under sustained load occurs. This too is a characteristic of local shear failure.

Definition of the ultimate pressure under such conditions is necessarily imprecise. Terzaghi states, "We specify arbitrarily, but in accordance with current conceptions, that the earth support has failed as soon as the curve passes into a steep and fairly straight tangent."

The construction involving the intersection of the initial elastic range and steep tangent extensions (see curves of q vs. S_B

following) is a definition given by Sowers (1962) which allows direct comparisons of data within the testing program.

Finally, the pressure at which shearing of the soil surface at the footing boundary was first noted, q_p, is defined as the *perimeter shear stress;* while to term *punching* refers to the vertical or near-vertical slip plane formation under the test footing. This latter condition has been noted in stiff soil over soft soil configurations tested by previous investigators (Tcheng, 1957; Koizumi, 1965). These researchers concerned themselves with the possibility of straight vertical punching through the top, stiff layer. However, Brown (1967) recognized the fact that, although this vertical punching may be initiated, it need not necessarily be completed before the soft layer beneath is sheared. This phenomenon is defined herein as "aborted shear punching."

4.2 Homogeneous Soil Tests

A footing test was performed on both the stiff and soft soil components in a homogeneous clay foundation. These two tests were intended only as an internal check against the layered soil tests of the program since, as pointed out in Section 1.3a, such homogeneous clay foundations have been thoroughly researched elsewhere.

The pressure-settlement curves for these two tests are illustrated in Figures 4-1 and 4-2. The corresponding values of N_m, the modified bearing capacity factor, are listed in Table 2, Summary of Footing Test Results, which follows. Since the subsoils of the two tests are not layered systems, N_m equals N_c. However, the experimentally

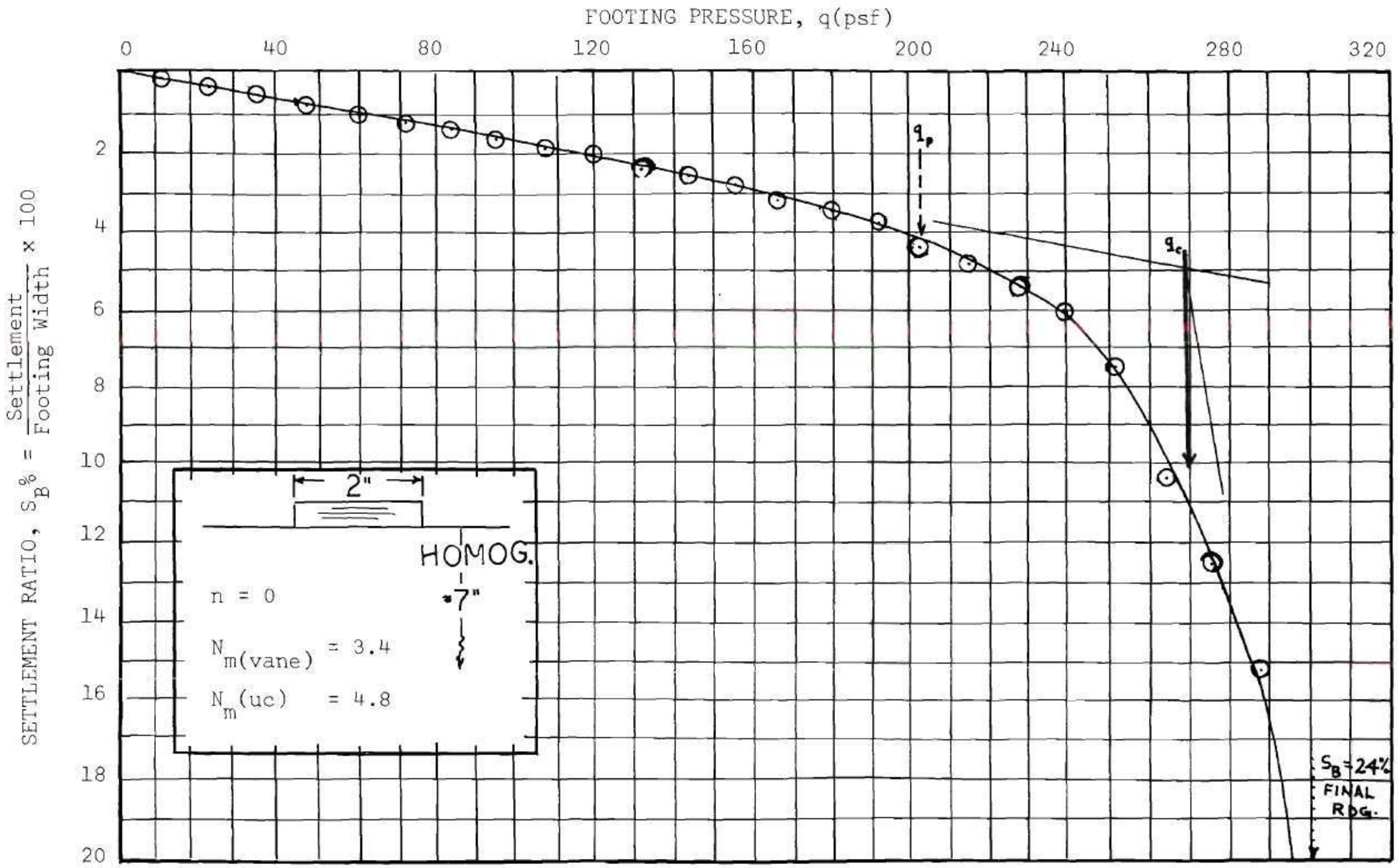

Figure 4-1. Pressure-Settlement Curve: Test 2HS-12, Homogeneous Stiff Soil

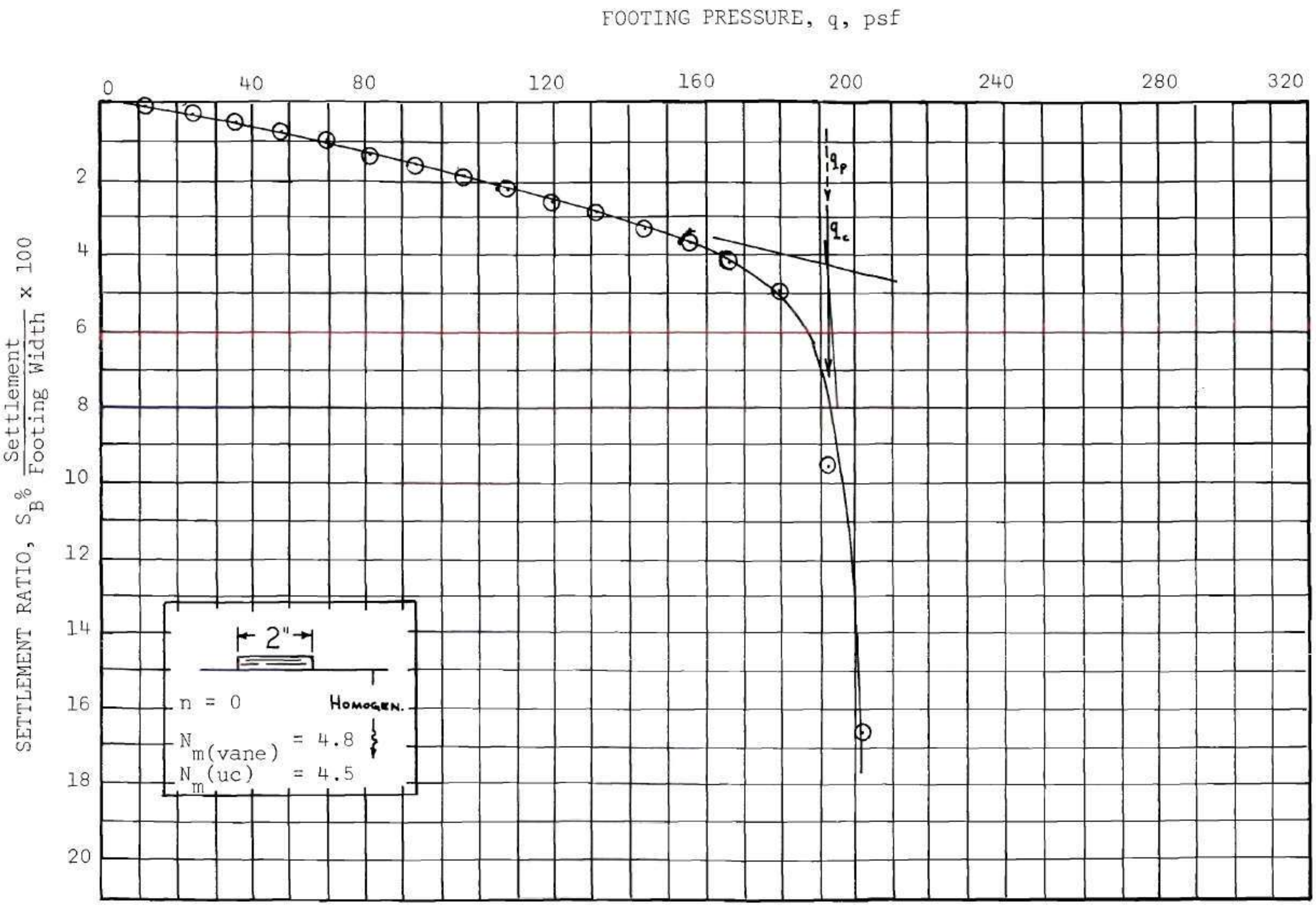

Figure 4-2. Pressure-Settlement Curve: Test 2HS-16, Homogeneous Soft Soil

determined factors are found to be lower than those predicted by general shear theories. In the first test (2HS-12) the overprediction averages at 33 per cent; it is 19 per cent for test 2HS-16. Such an observation is consistent with the assumption of local shear failure.

As noted in Figures 4-1 and 4-2, the values of S_B (settlement in percentage of footing width) at failure are on the order of 10 per cent. However, it should be noted that the system deflects elastically for much of this range, so that the problem of footing penetration prior to failure, mentioned in Chapter II, is mitigated a great deal. The true footing penetration is indicated roughly by the spread between q_p and q_c on the vertical axis of the pressure-settlement curves.

The differences in stress-strain properties* are well pointed out in these two tests. It was a characteristic of all tests involving a thick top layer of the stiffer soil (as test 2HS-12) that the clay topsoil exhibited a broad transition region on the pressure-settlement curve. This makes definition of bearing capacity that much more open to question in such cases.

4.3 Stiff Layer Over Soft Layer Soil Systems

4.3a Thin Stiff Top Layer (d/b = 0.5)

This is perhaps the most commonly encountered situation in field practice, i.e., a case where a thin stiff crust overlies a deep softer deposit. Six model tests were performed with this subsoil

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*Stress-strain curves are presented in Appendix B. These curves are each the average values of unconfined compression tests performed for the particular footing test noted.

Table 2. Summary of Footing Test Results*

Test No.	d/b	q_c (psf)	S_B% at q_c	n vane	n UC	N_m vane	N_m UC	N_m, Related Methods (Using Average n value) of Theory (See Appendix A)
2HS-12	Stiff	268	11.0	0	0	3.4	4.8	Fellenius, 5.5; Prandtl, 5.14
2HS-16	Soft	192	7.8	0	0	4.8	4.5	As above
2LS-1	0.5	180	9.0	-0.62	-0.63	2.25	2.25	Nixon, 2.32; Button, 2.64; Brown (Aborted Punching), 2.32
2LS-2	0.5	182	8.1	-0.62	-0.63	2.27	2.26	As above, 2LS-1
2LS-5	0.5	213	6.3	-0.55	-	2.67	-	Nixon, 2.79; Button, 3.18; Brown (Aborted Punching), 2.65
2LS-6	0.5	209	8.7	-0.55	-	2.62	-	As above, 2LS-5
2LL-21	0.5	137	5.0	-0.24	-0.50	2.37	2.94	Nixon, 3.95; Button, 4.0, Brown (Aborted Punching), 3.66
2LB-22	0.5	183	7.2	-0.43	-0.44	1.97	2.37	Nixon, 3.35; Button, 3.60; Brown (Aborted Punching) 3.13
2LS-3	1.0	248	10.0	-0.40	-	3.10	-	Nixon, 4.36; Button, 3.85; Brown (Aborted Punching), 3.84
2LS-4	1.0	240	7.5	-0.40	-	3.00	-	As above, 2LS-3
2LS-9	1.0	192	9.8	-0.33	-	2.40	-	Nixon, 4.86; Button, 4.30; Brown (Aborted Punching) 4.19
2LS-17	2.0	228	6.0	-0.27	-0.30	4.15	4.15	Button, 5.52; Brown (Aborted Punching), 5.14
2LB-19	2.0	210	7.5	-0.37	-	2.84	-	Button, 5.4; Brown (Aborted Punching) 4.77
2LS-7	3.0	254	11.5	-0.09	-0.06	3.44	5.12	Button, 5.52; Terzaghi local shear, 3.4; Brown, 5.14
2LS-7	3.0	369	14.1	-0.17	-0.30	4.40	5.85	As above, 2LS-7
2LB-11	3.0	292	7.8	-0.31	-	3.22	-	As above 2LS-7
2LS-10	0.6	154	6.8	+0.53	+0.59	5.91	6.10	Button, 6.7; Brown (Test Data, Figure 1) 5.75; Tcheng-Prandtl, 7.6; Meyerhof and Chaplin, 5.83
2LS-13	0.6	197	7.9	-	1.16	-	7.6	Button, 7.3; Brown (Test Data), 5.52
2LS-14	0.6	213	8.5	+0.34	1.1	6.62	-	Button, 6.60; Brown (Test Data), 5.52
2LB-18	0.6	179	7.3	-	+0.6	-	5.3	Button, 7.3; Brown (Test Data), 5.52; Meyerhof and Chaplin, 5.83
2LL-20	0.6	200	5.7	+0.52	+0.14	3.95	4.22	Button, 7.3; Brown (Test Data), 5.52
2LS-15	0.8	207	6.8	+0.20	+0.18	5.05	4.20	Button, 6.1; Brown (Test Data), 5.1; Tcheng-Prandtl, 5.65
2LL-23	0.8	183	10.5	+0.64	-	4.35	5.22	Button, 6.1; Brown (Test Data), 5.40

*The notation used is that defined in the Glossary of Symbols, page vii.

configuration; the pressure-settlement curves are shown in Figures 4-2 to 4-8. It should be pointed out that (referring to Table 2) tests 2LS-1 and 2LS-2 and also 2LS-5 and 2LS-6 were performed on the same test soil system; that is, two footing tests were originally performed in one of the test boxes. As shown in Table 2 this procedure had little effect on the ultimate bearing capacity, but it does change the slope of the pressure-settlement curves in the elastic region. Remaining tests were therefore limited to one test per box.

It is seen from Table 2 that the calculated values of N_m for the d/b = 0.5 tests are in all cases lower than those predicted by the existing solutions; none of these are conservative. The theoretical solutions give the best agreement where the strength difference between the two component layers is greatest (as in test 2LS-1). Button's method in all cases was the most unsafe estimate, varying roughly from 10 per cent to 45 per cent overprediction.

The Nixon load-spread technique is definitely not a conservative estimate for the test cases, although for the cases (2LS-1,2,5, and 6) of $n \leq -0.55$ it is an increasingly good approximation; e.g., the overestimates for test 2LS-1 and 2LS-2 are only about 3 per cent. This is in agreement with the discussion of prior section 1.3b.

Finally, Brown's punching solution gives the best correlation with test data. The range is from near-coincidence to approximately 4 per cent overprediction for tests 2LS-1, 2, 5, and 6.

Test 2LB-22 was a split-box test which afforded an opportunity to observe the failure plane formation at the outset of rapid settlement. Figure 4-8 indicates the point at which load was removed.

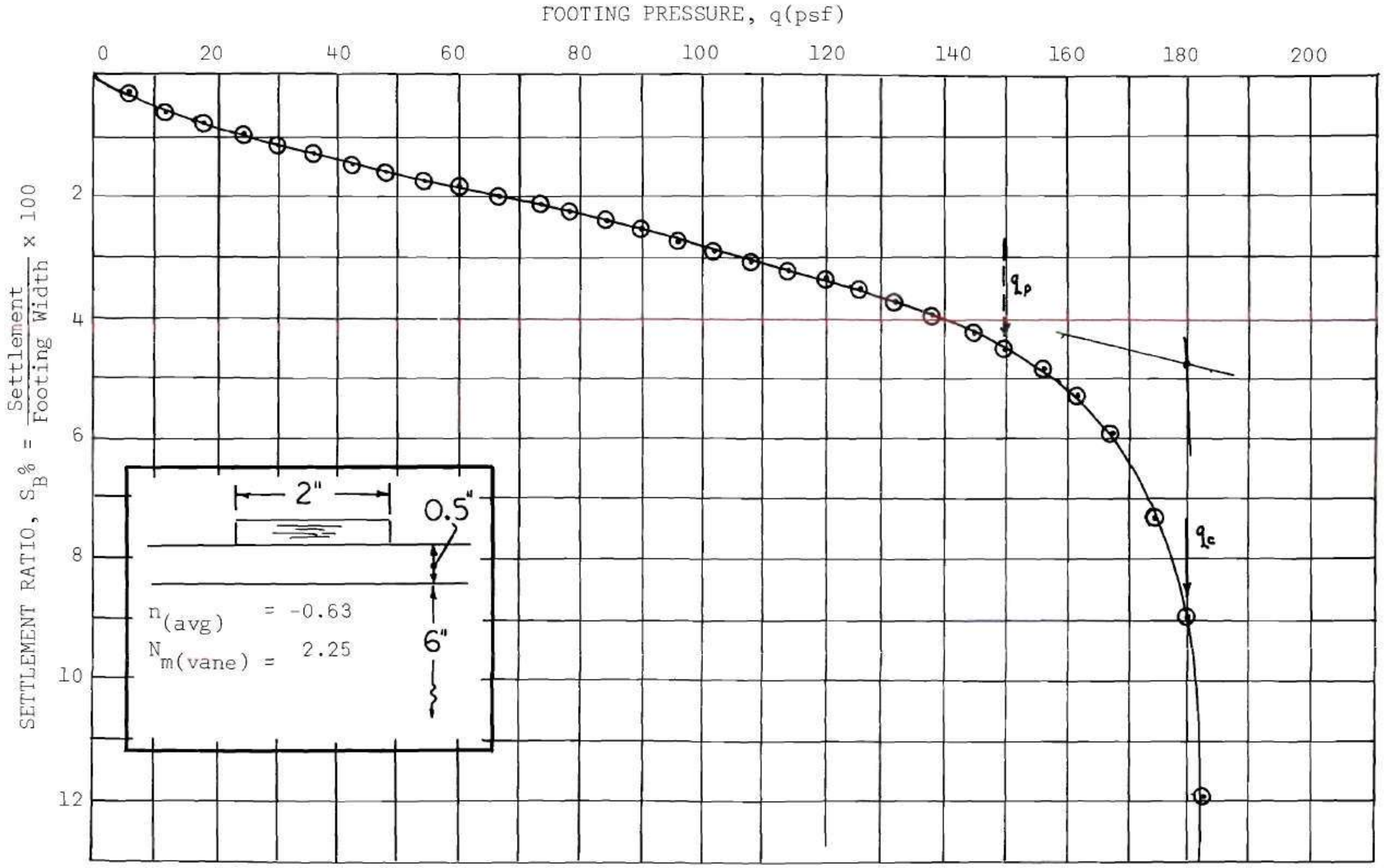

Figure 4-3. Pressure-Settlement Curve: Test 2LS-1, d/b = 0.5, Stiff Layer over Soft Layer Configuration

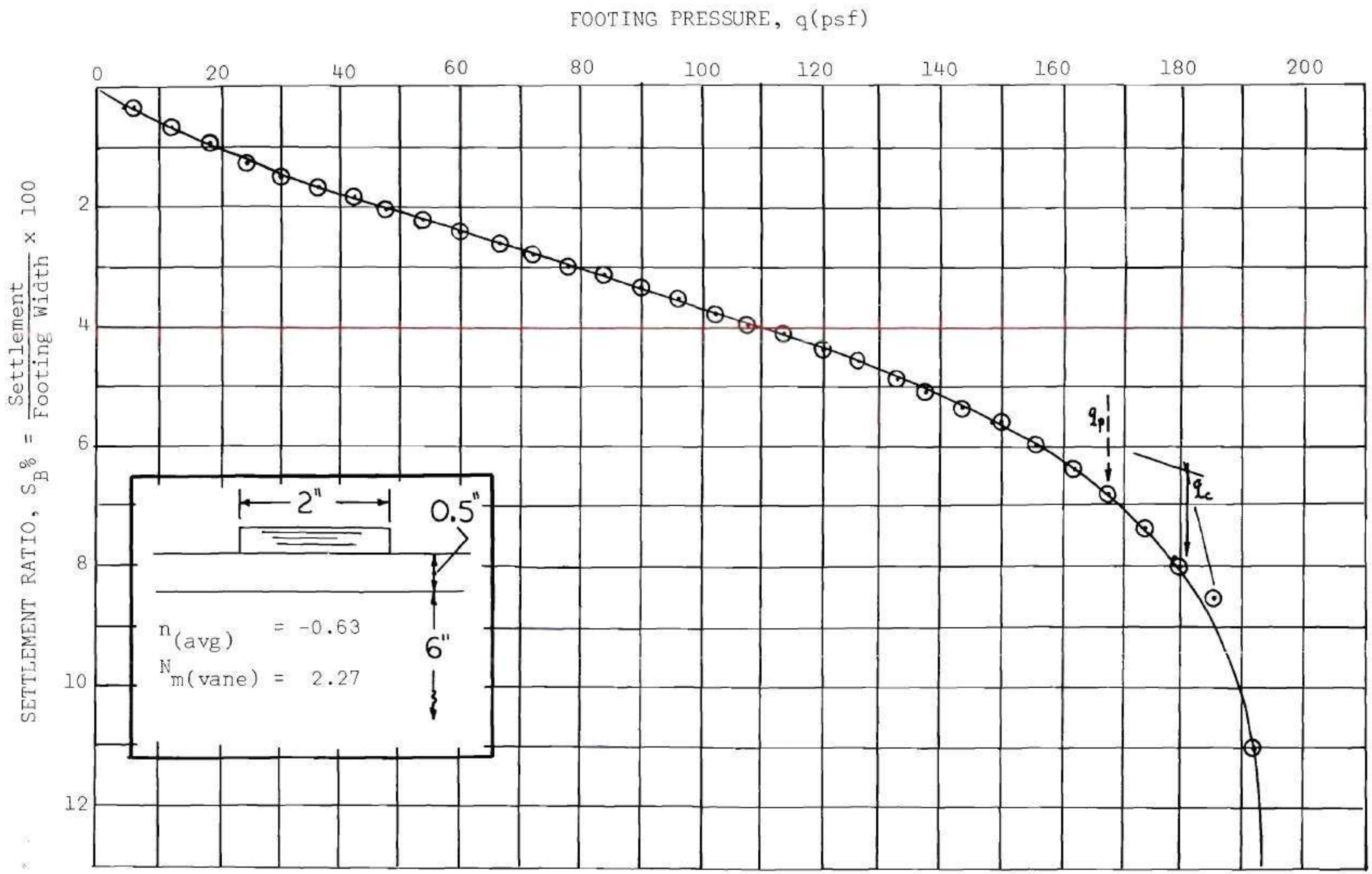

Figure 4-4. Pressure-Settlement Curve: Test 2LS-2, d/b = 0.5, Stiff Layer over Soft Layer Configuration

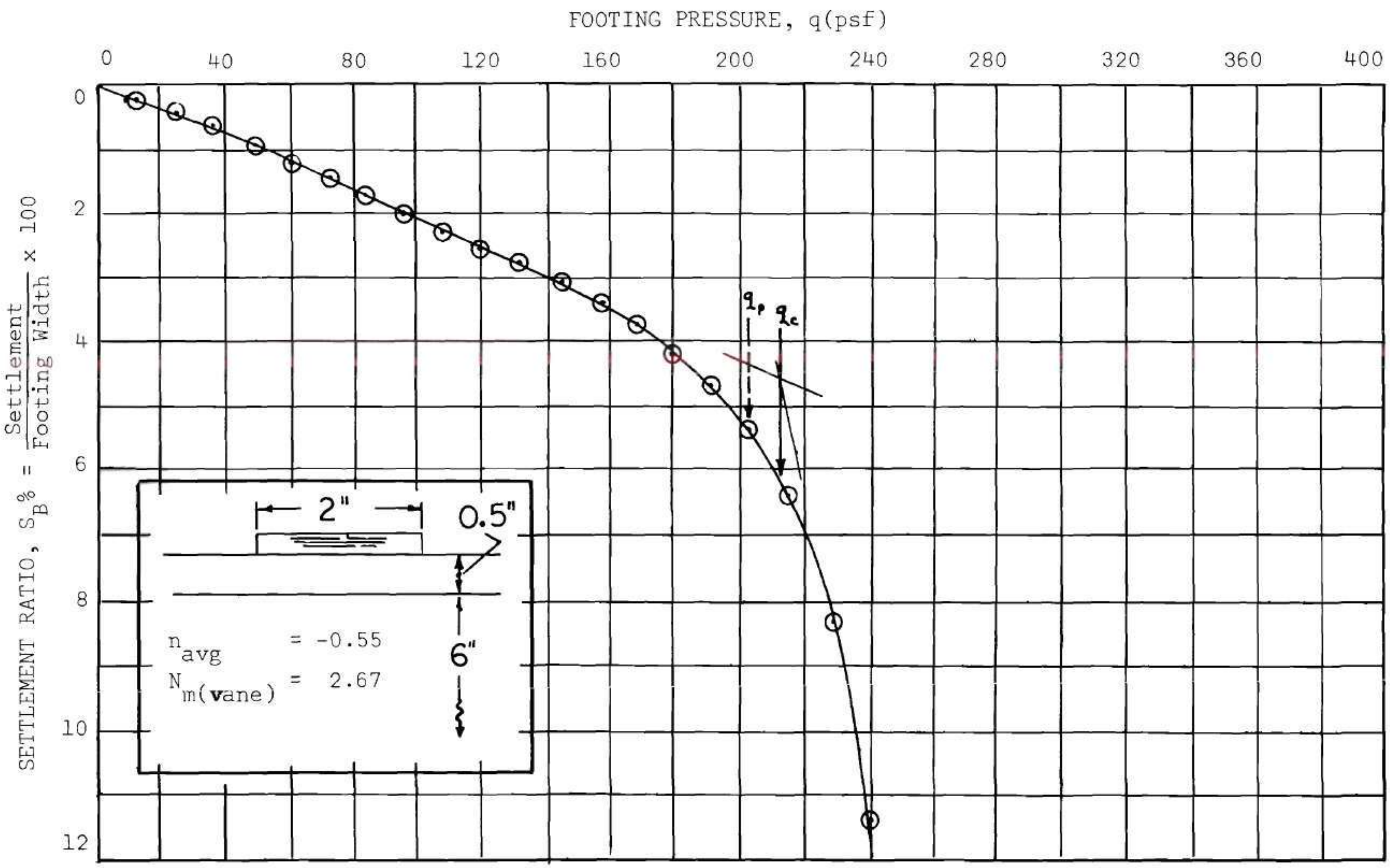

Figure 4-5. Pressure-Settlement Curve: Test 2LS-5, d/b = 0.5, Stiff Layer over Soft Layer Configuration

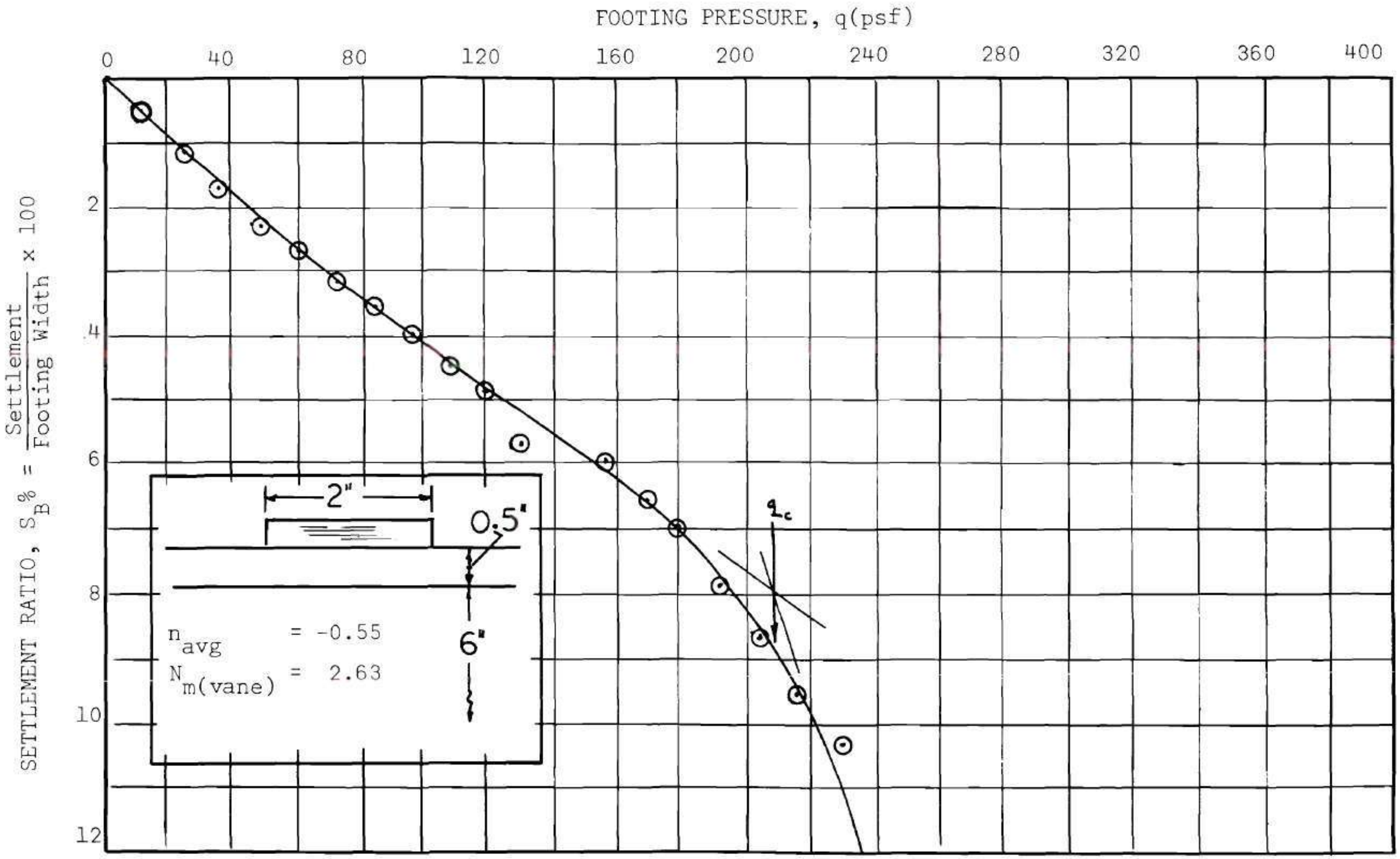

Figure 4-6. Pressure-Settlement Curve: Test 2LS-6, d/b = 0.5, Stiff Layer over Soft Layer Configuration

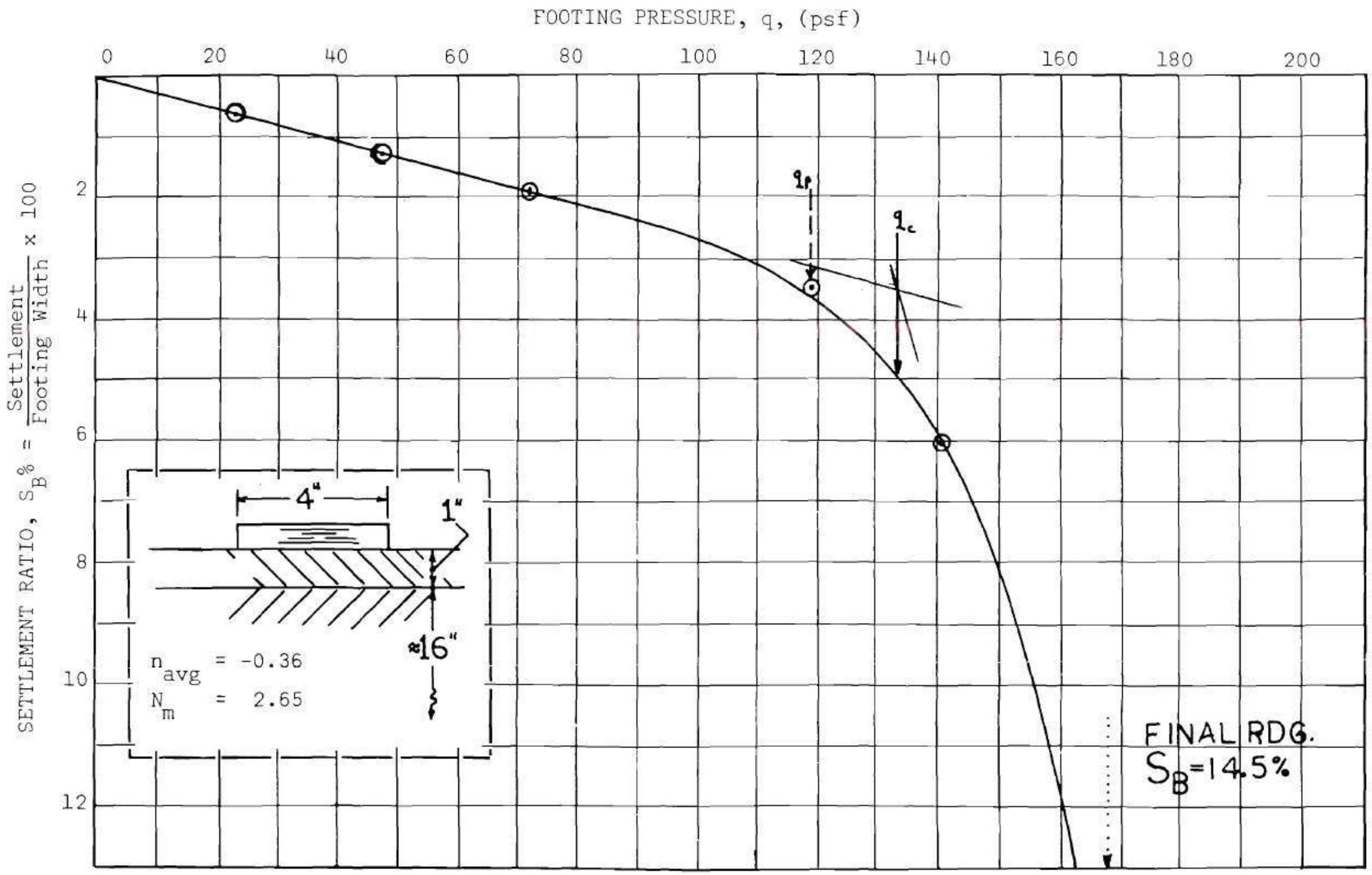

Figure 4-7. Pressure-Settlement Curve, Test 2LL-21, d/b = 0.5, Stiff Layer over Soft Layer Configuration

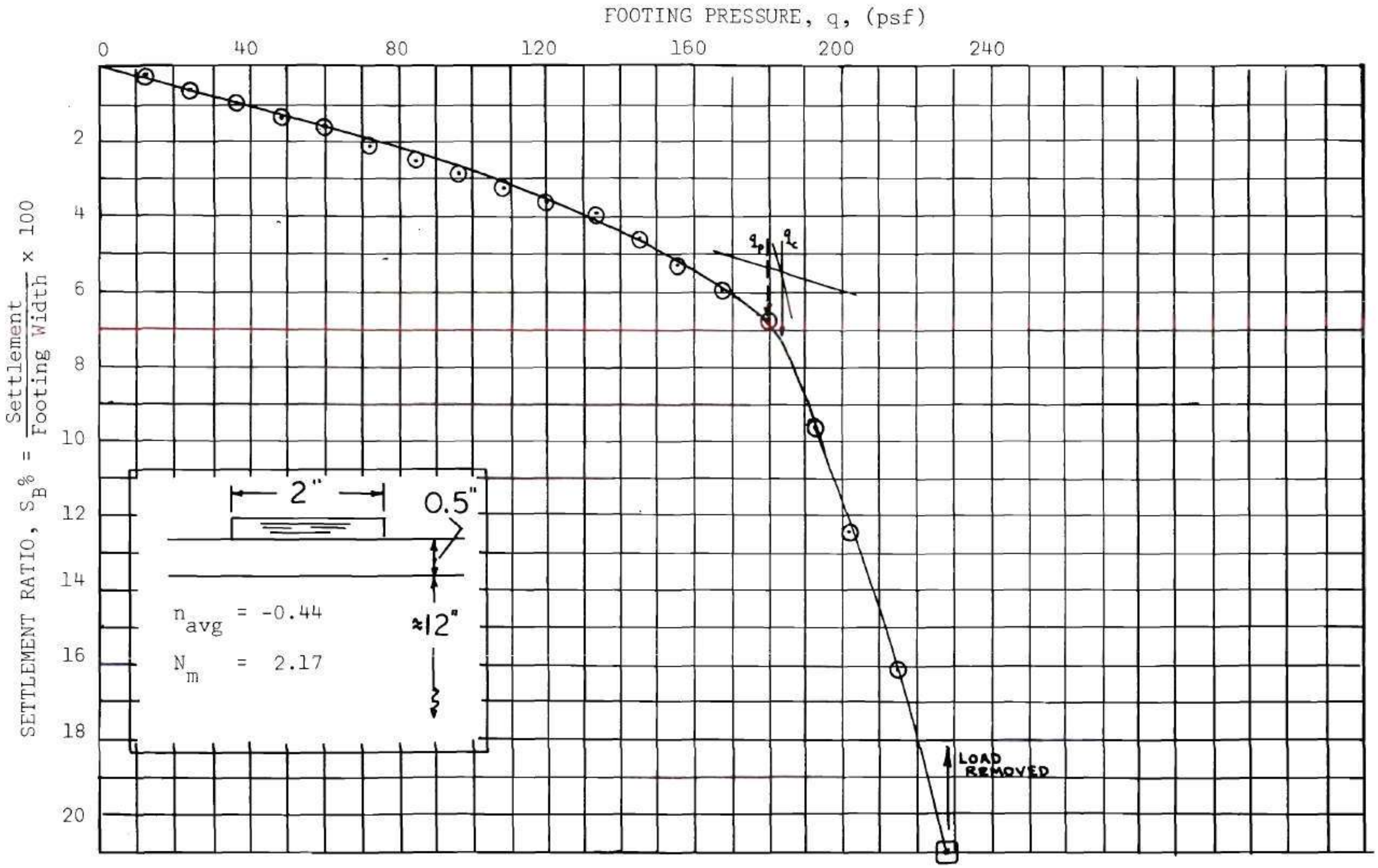

Figure 4-8. Pressure-Settlement Curve, Test 2LB-22, d/b = 0.5, Stiff Layer over Soft Layer Configuration

A photo of the failure is presented as Figure 4-9, with a line sketch (Figure 4-10) of the original photo following. Referring to Figure 4-10, the following interpretation of events is given.

Perimeter shear punching, 1 , occurred at pressure of approximately q_p (see Figure 4-8). The wedge under the footing, 2 , appears to be a "dead" elastic one, extending without noticeable discontinuity at the layer boundary. The shear planes 3 diverging from the apex of the wedge do not spread beyond the footing width. It seems that the soil was undergoing essentially elastic compression outside this zone; however, evidences of shear planes 4 outside the footing width were noted. They seemingly have a contradictory orientation, not aligning with the rest of the shear pattern. The following explanation is proposed.

As noted from Figure 4-10, there is a slight bulge 5 on both sides of the failed footing, this bulge is (approximately) repeated at the layer interface. Therefore, as the footing settled, most of the distortion was experienced in the soft lower layer. Since the lower layer is of very low strength ($c_2 \approx$ 44psf), it would seem that the Terzaghi-Bell mechanism (Sowers and Sowers, 1961) was operative; i.e., the soft soil outside the footing width is analogous to an unconfined compression test in horizontal loading. This can also be used to explain the orientation of the planes at 4 , since in an unconfined compression test which involves eccentric loading shear planes will be forced in one direction. The compression of course will be progressively less with depth in the footing test. Therefore, these planes at 4 will be forced downward rather than up toward the surface.

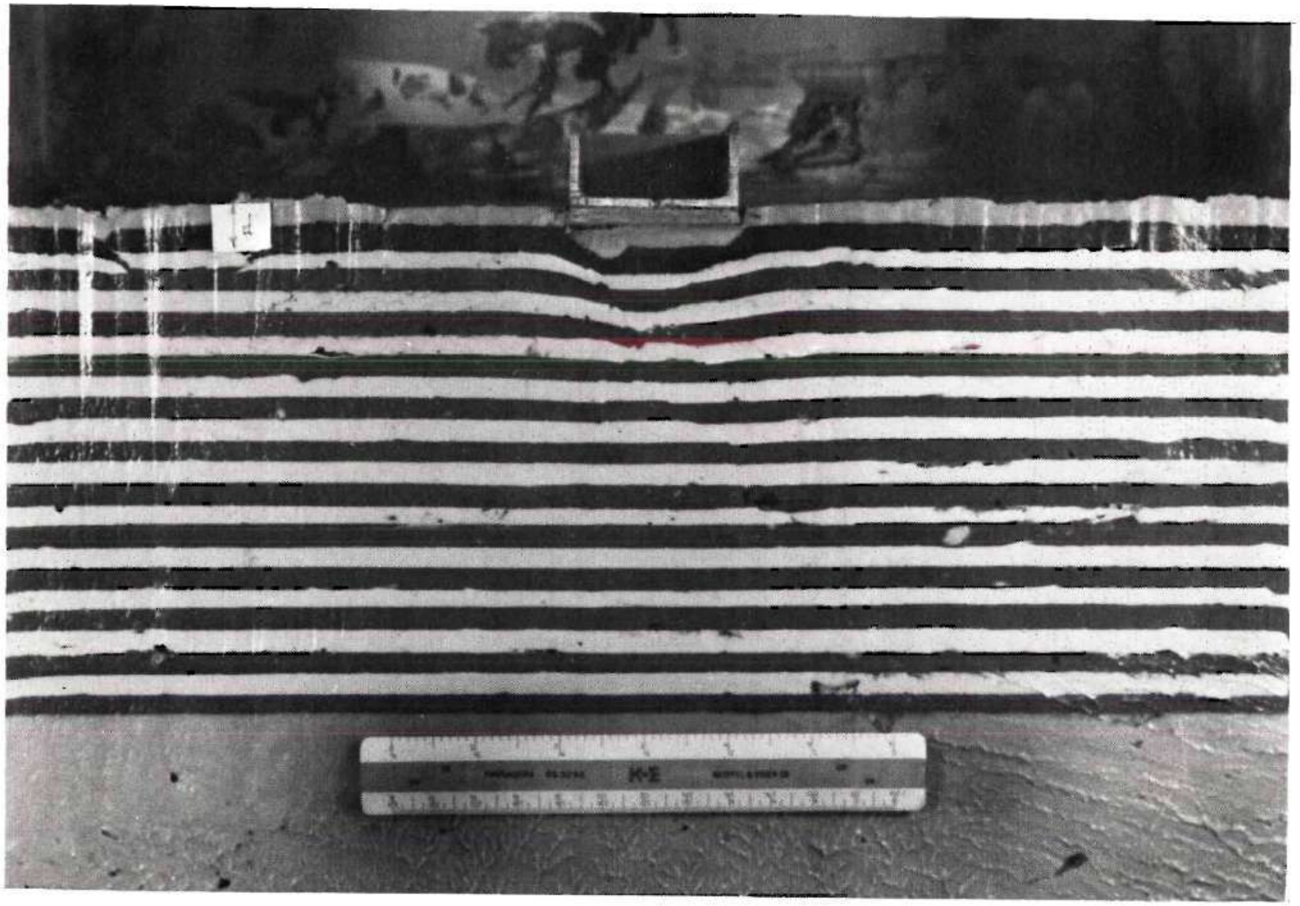

Figure 4-9. Failure Conditions, Test 2LB-22, d/b = 0.5, n = -0.44

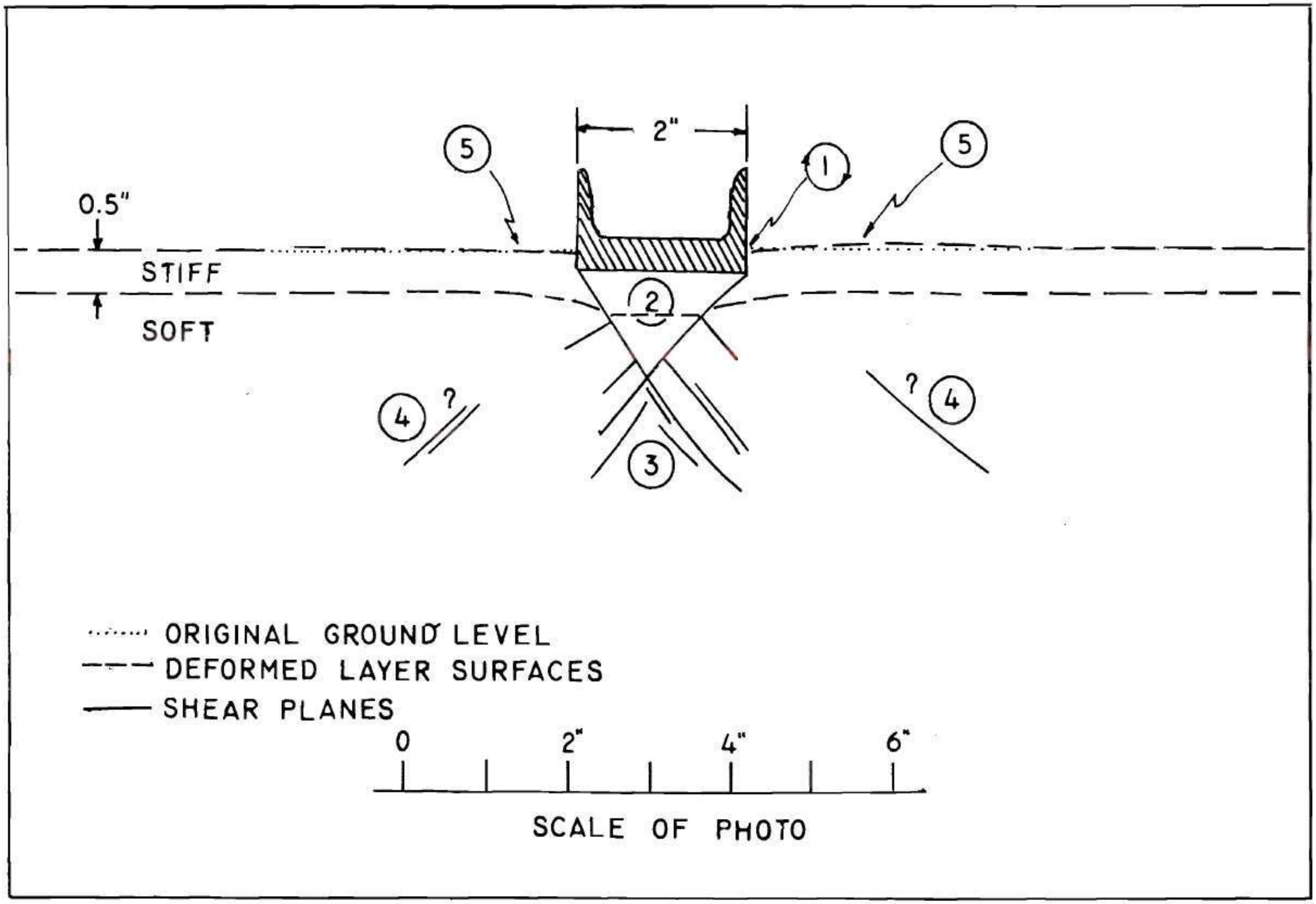

Figure 4-10. Failure Conditions, Test 2LB-22, d/b $\doteq$ 0.5, n = -0.44

Because a great deal of lateral strain is required to develop this condition in a compressible clay, it no doubt occurs after bearing capacity failure.

4.3b Stiff Layer of Intermediate Thickness (d/b = 1.0; d/b = 2.0) Overlying Soft Soil

Five tests were performed in this set, three (tests 2LS-3, 2LS-4; 2LS-9) were with a d/b = 1.0, while the remaining two (2LS-17 and 2LB-19) involved d/b = 2.0.

Again referring to Table 2, it was found that the load spread solution is no longer even approximately true for such thickness of layers. Button's solutions for these cases, unlike those of the previous (d/b = 0.5) discussion, give results which are closer to the experimental values. However the overpredictions are still very high (24 to 52 per cent).

Although it is unjustified to draw conclusions on the basis of a single test, the test 2LS-17 (Figure 4-14) is of particular interest since it is predicted by the theories to be one where bearing capacity is completely governed by the top layer strength. That is to say, the loaded footing does not "sense" the soft lower layer. If this is so, then an internal check of the data is possible. Test 2HS-12, the homogeneous stiff soil test, yielded a vane strength c = 80 psf and average $N_m = 4.1$. The above test 2LS-17 yielded a vane strength of top stiff layer $c_1 = 74$ psf and an $N_m = 4.15$. The two N_m values are in approximate agreement, but there is still a considerable discrepancy with the $N_m = N_c = 5.14$ to 5.52 given by the homogeneous soil theories. However if the one-third reduction proposed by Terzaghi (1943) for local

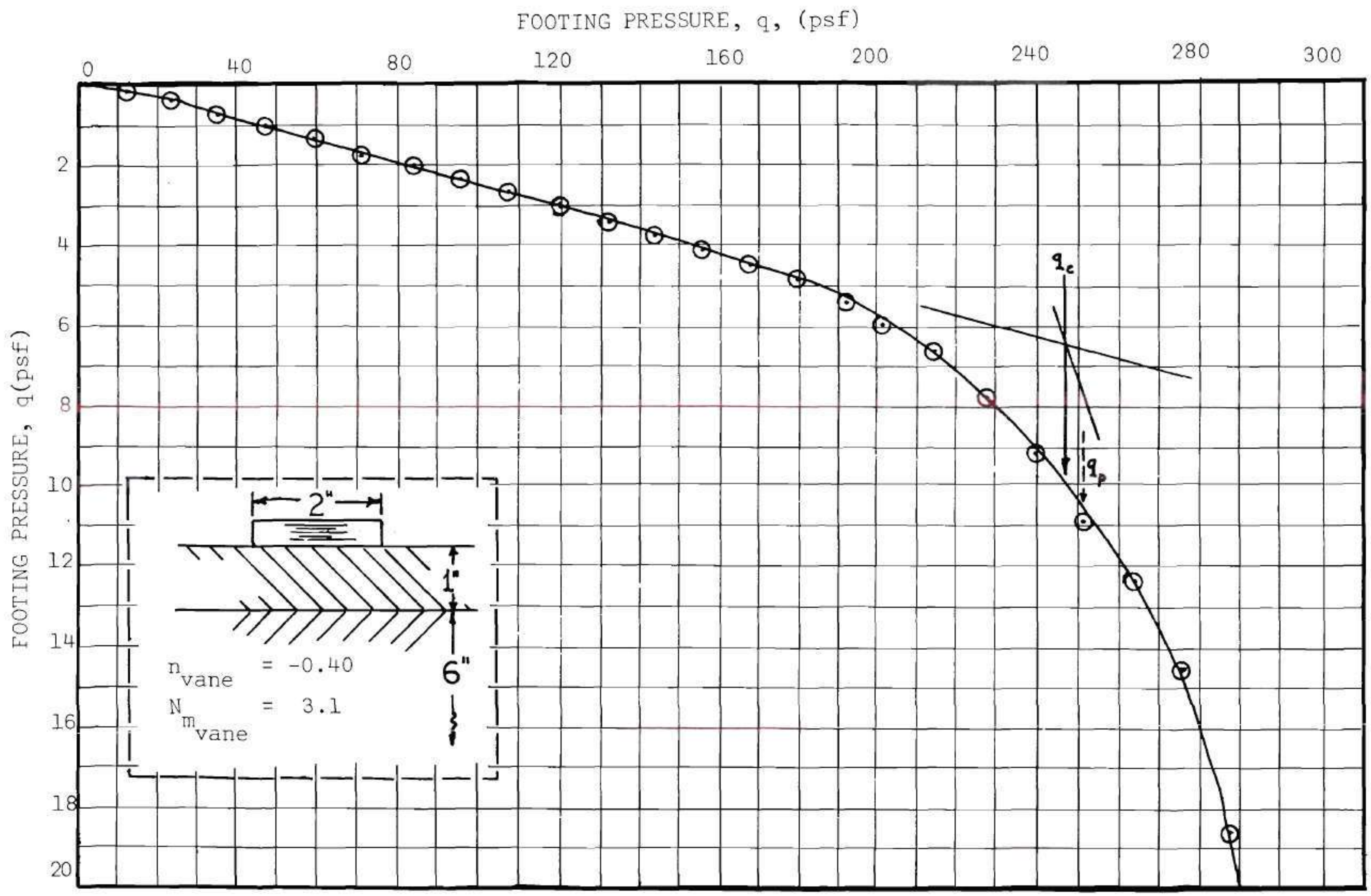

Figure 4-11. Pressure-Settlement Curve: Test 2LS-3, d/b = 1.0, Stiff Layer over Soft Layer Configuration

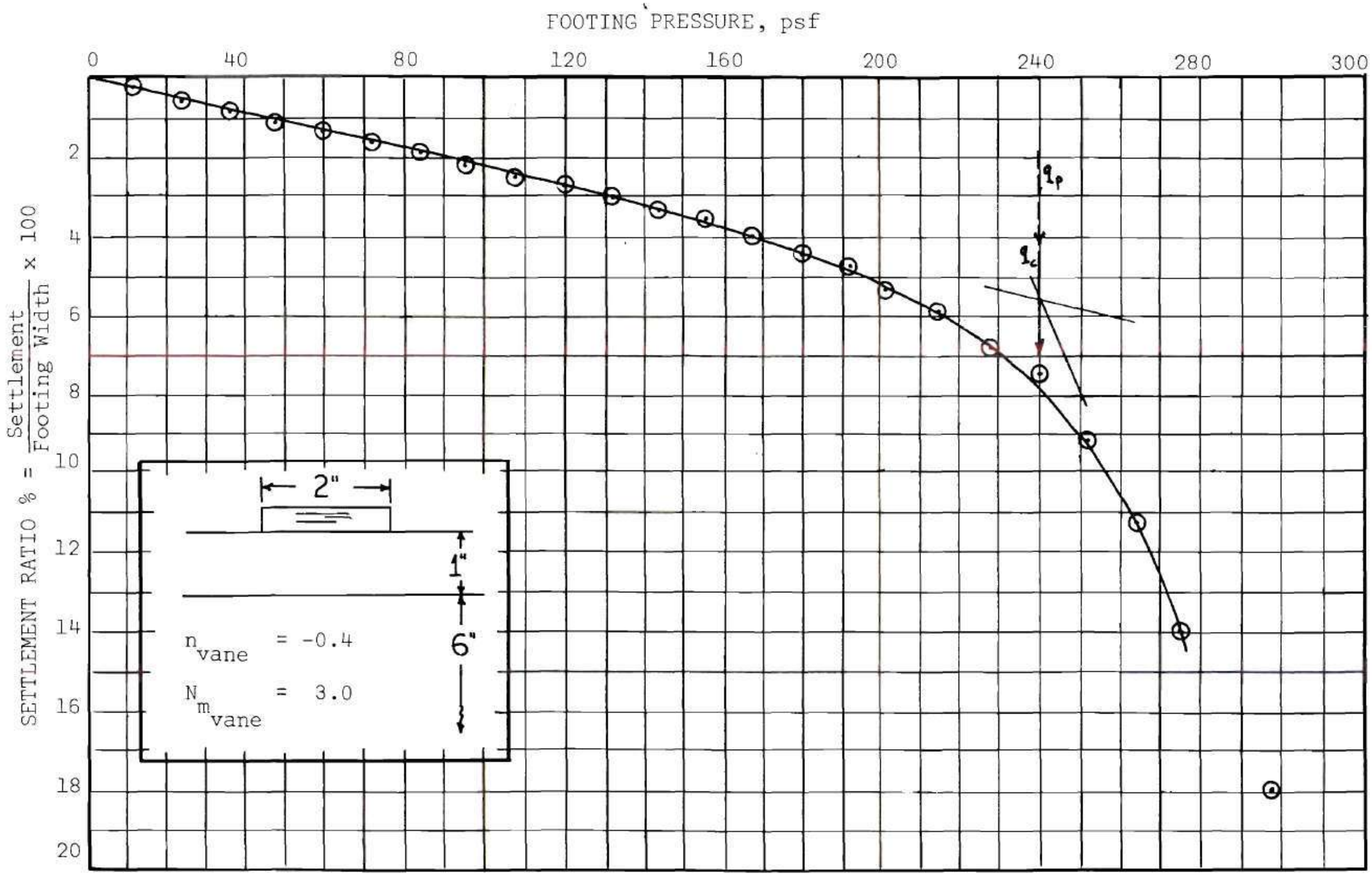

Figure 4-12. Pressure-Settlement Curve: Test 2LS-4, d/b = 1.0, Stiff Layer over Soft Layer Configuration

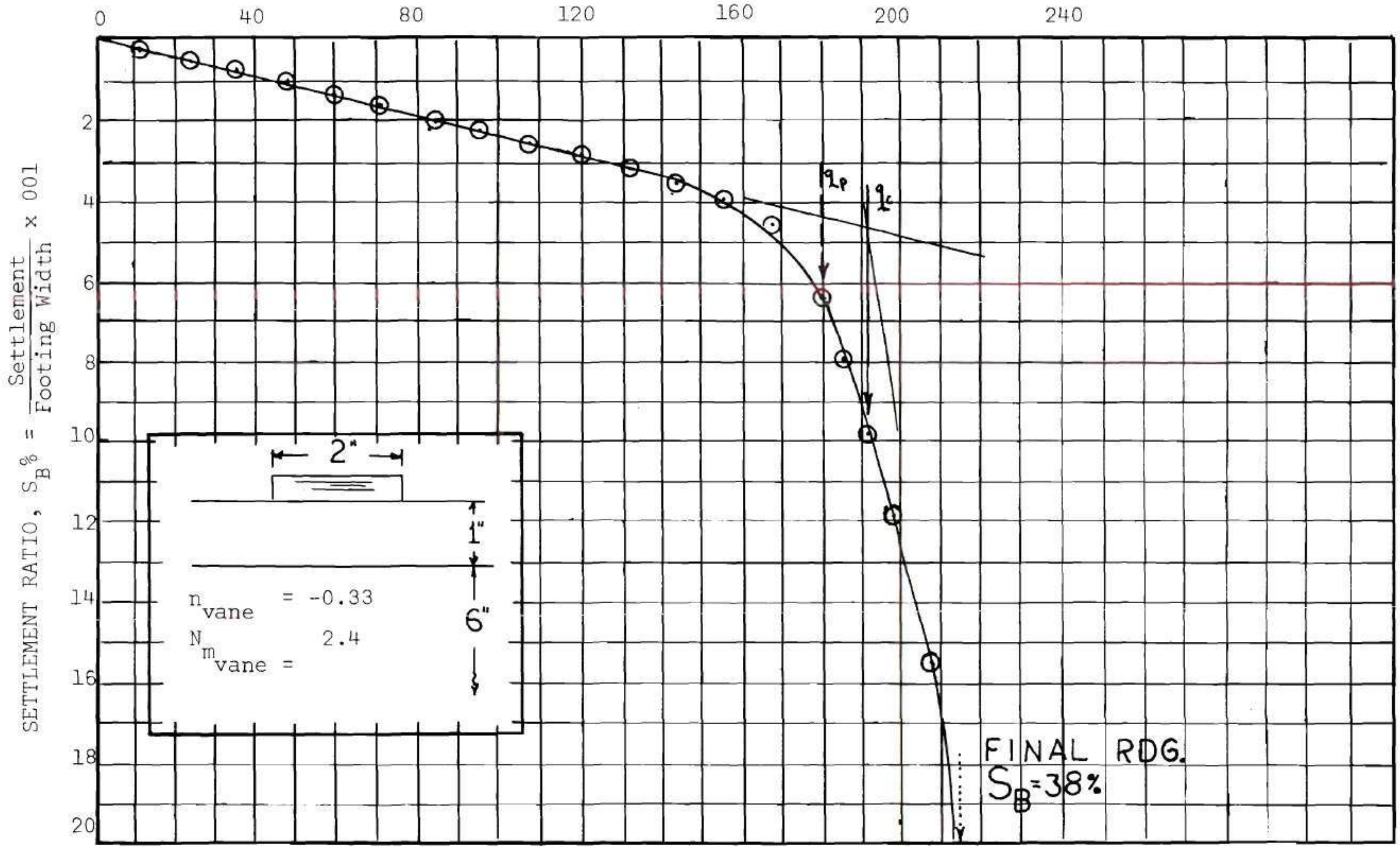

Figure 4-13. Pressure Settlement Curve: Test 2LS-9, d/b = 1.0,
Stiff Layer over Soft Layer Configuration

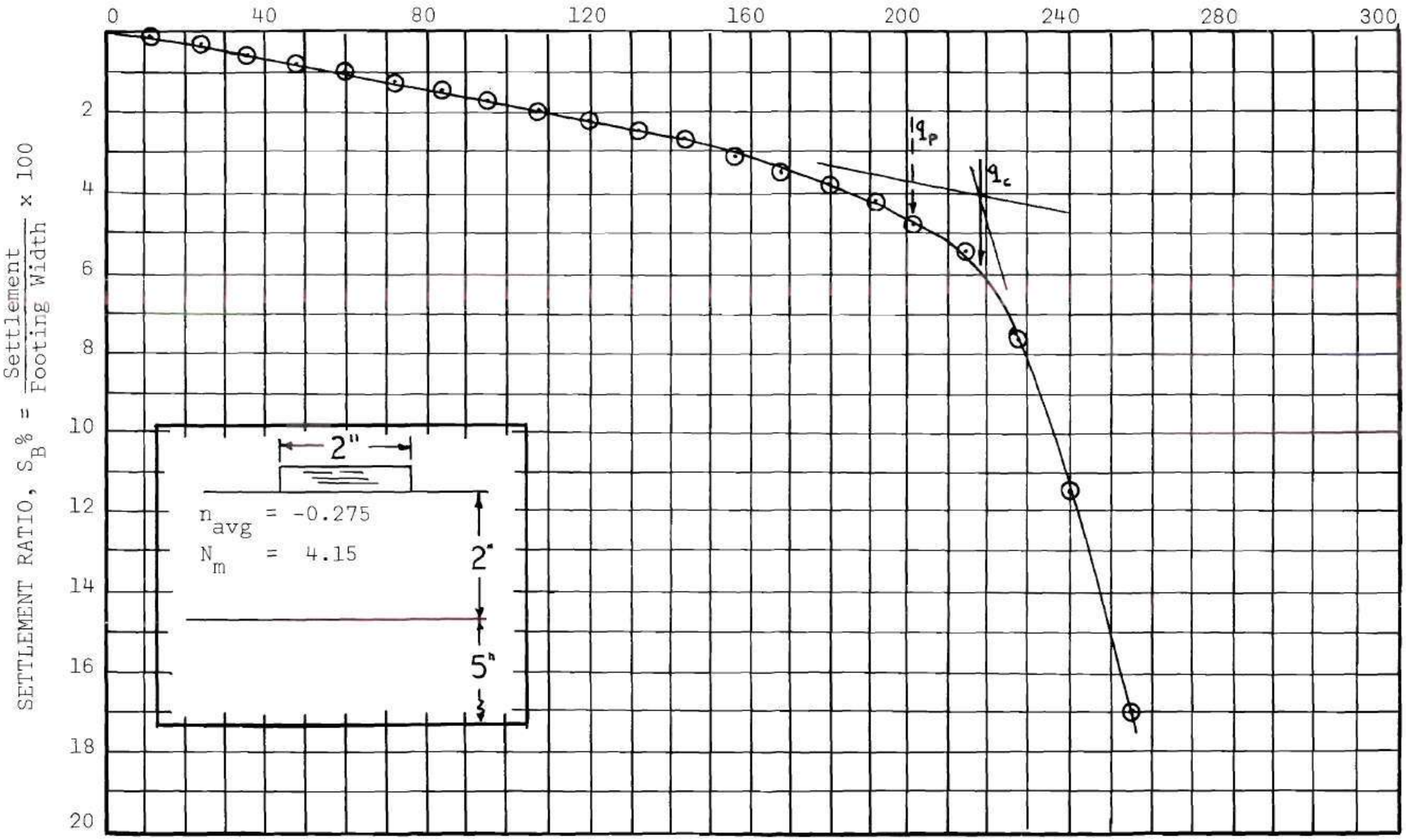

Figure 4-14. Pressure-Settlement Curve, Test 2LS-17, d/b = 2.0, Stiff Layer over Soft Layer Configuration

shear is applied, the theoretical value checks with the experimental $N_m = 4.15$.

A split box test was also performed for this set of tests ($d/b = 2.0$) and is recorded in Table 2 as test 2LB-19. Figure 4-15 indicates the point at which load was removed; the corresponding illustrations are noted in Figures 4-16 and 4-17. These latter two figures show that the wedge under the footing is confined to the upper layer; in fact, its apex is approximately at the layer interface. Even discounting the $\phi = 0$ assumption of 45° slip planes in favor of planes governed by the true friction angle ($45° + \phi_e/2$) certain observations could not be readily explained. As indicated by numeral 1 of Figure 4-17 there is some evidence of zone shear below the footing edges which does not align with the distinct lines of the wedge itself. Furthermore, there is some indication, 2 , that the wedge is not a dead elastic one.

An explanation may be as follows: At a certain value of pressure q less than the eventual ultimate, a wedge formed with its apex in the vicinity of 2 . As the pressure increased a deeper zone of soil was of course stressed and a wedge reformed, allowing shear planes at 3 to form in the much softer ($n = -.28$) bottom layer, rather than continuing shear planes through the stiff material at 2 .

Such an explanation involves a distinct departure from the theoretical solutions previously discussed since it admits the possibility of multiple, discontinuous slip planes. Because such a condition

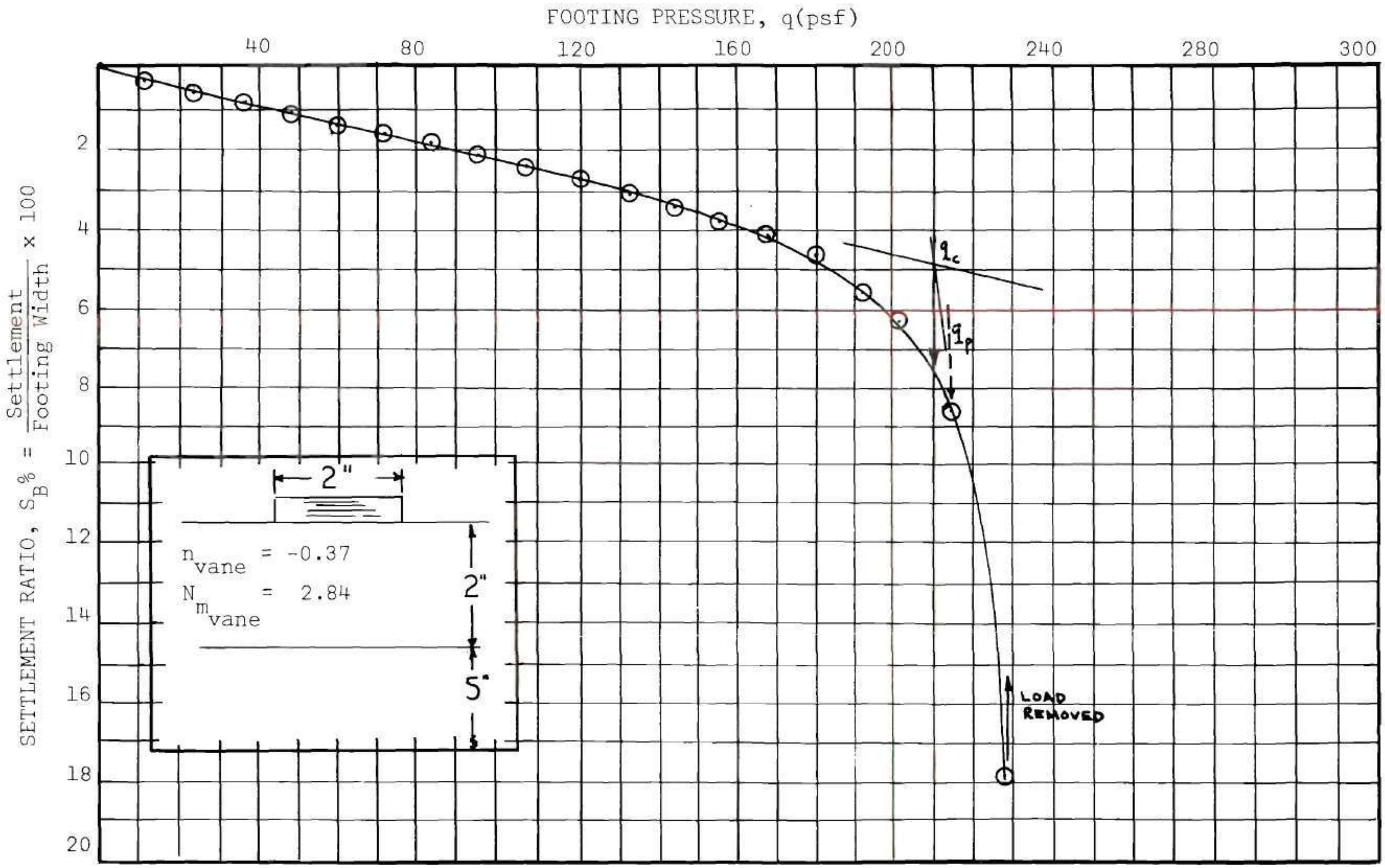

Figure 4-15. Pressure-Settlement Curve, Test 2LB-19, d/b = 2.0, Stiff Layer over Soft Layer Configuration

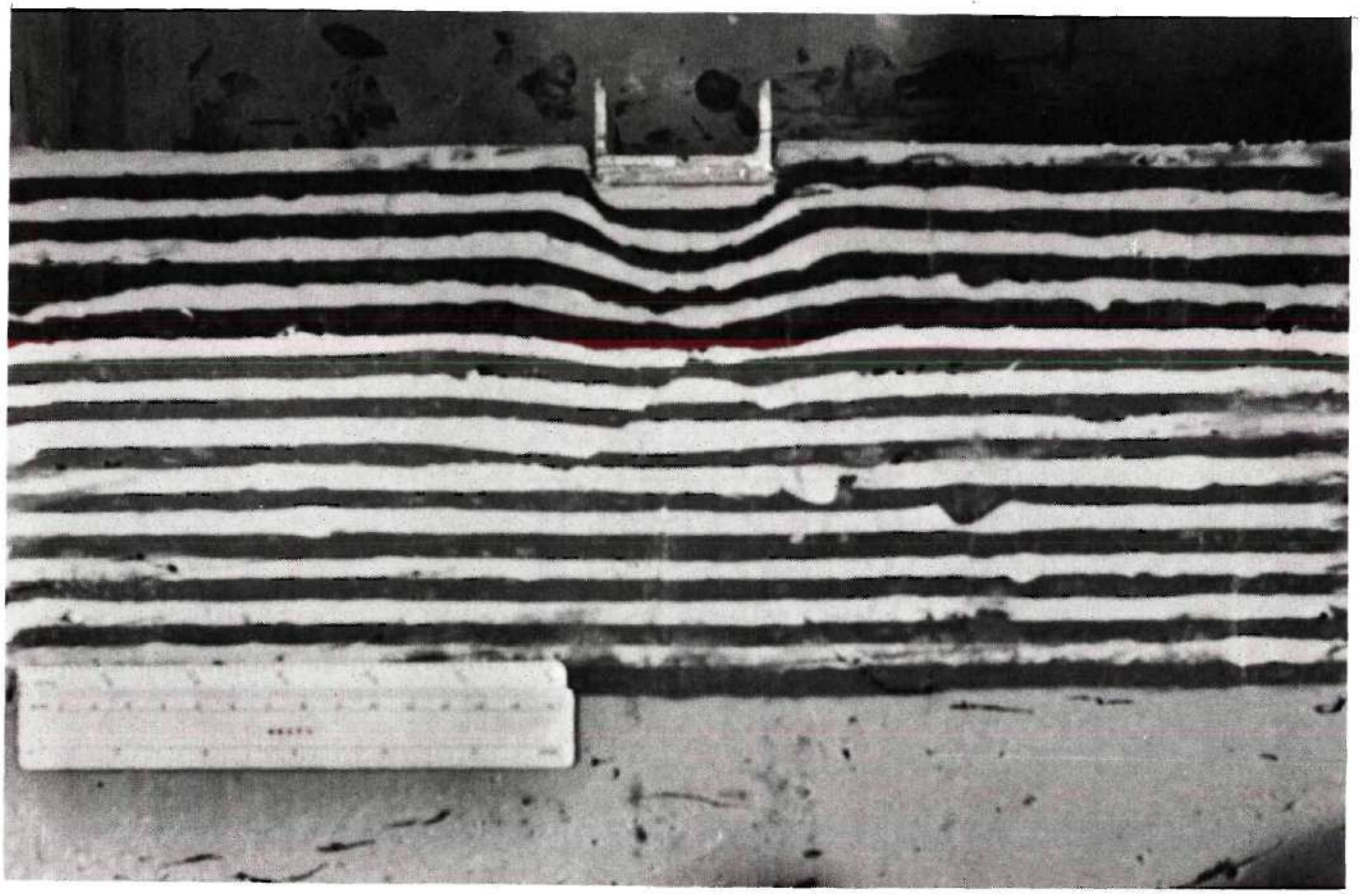

Figure 4-16. Failure Conditions, Test 2LB-19, $d/b = 2.0$, $n \doteq -0.28$

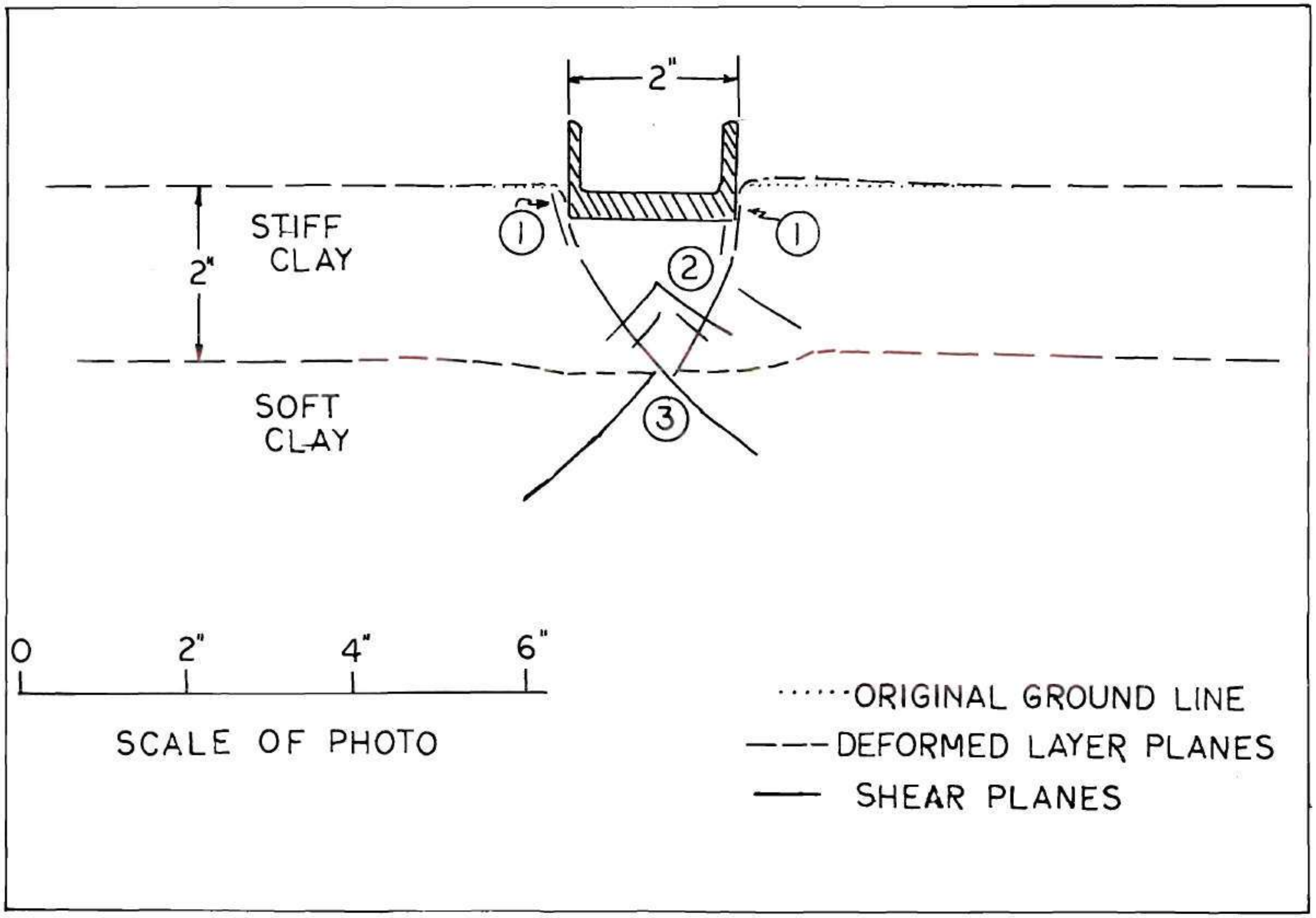

Figure 4-17. Failure Conditions, Test 2LB-19, d/b = 2.0, n ≐ -0.28

is not considered*, calculated bearing capacities are likely to be too high, even where local shear failure is not a consideration.

4.3c Thick Stiff Layer (d/b = 3.0) Overlying a Soft Layer

Three tests were performed in this set (tests 2LS-7, 2LS-8 and 2LB-11). As indicated by Table B-1 (Appendix B) the vane-UC ratios were very high for this particular set. This makes quantitative interpretations subject to much uncertainty. The problem is brought out in the results of test 2LS-7, Figure 4-18, where the solution using vane results gives near-coincidence with a homogeneous local shear analysis, while the solution using unconfined compression data yields close agreement with Brown's solution which is based on general shear. The true answer is likely between the two solutions. With the data taken it is not possible to conclude where the answer lies in that range. For this set the discussion will therefore be concentrated on the qualitative results of the split box test 2LB-11 (d/b = 3.0).

Referring to Figures 4-21 and 4-22, as noted by numeral 1 of the latter figure, the footing rotated under load; the wedge at 3 is therefore decidedly skewed. It appears that failure is confined to the upper layer, since those incipient planes at 4 are associated with a strain considerably greater than that of S_B at failure (see Figure 4-20). The illustrations of Figure 4-23 and 4-24 were obtained by clamping an oiled glass pane against half of the split footing box and then reloading the footing half to very large strains.

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*The complexity of a solution which does consider this possibility is pointed out by Salas, J.A.J. (1965), "Discussion," *Proc.* Sixth Int. Conf. on S.M. and F.E., Vol. 3, pp. 415-418.

The conditions represented by Figures 4-23 and 4-24 are associated with strains well beyond that at bearing capacity; it is intended therefore to describe qualitatively the sequence of events as the footing penetrates deeply into the stiff top layer.

As shown in Figure 4-24, the rotation of the footing, (1) , was quite pronounced. The wedge at (2) has undergone severe plastic distortion, the bowl-shaped bands of the wedge in Figure 4-23 indicate that there was plastic flow of the wedge material out from under the footing. Another wedge of stiff material is noted to have reformed immediately under the distorted area. At the layer interface, (3) , there is an apparent distontinuity of shear planes. The planes diverge widely into the soil bottom layer (5) , but rather than recurving back upward through the stiff top material, a series of slip line "chevrons" formed (4) --again characteristic of a local shear failure.

4.4 Conclusions and Recommendations: Cases of Stiff Layer over Soft Soil

1. For soft compressible clays in a two-layer system, local shear conditions prevailed; all of the existing solutions, which were based on general shear, yield values of bearing capacity on the unsafe side.

2. Brown's shear punching solution,

$$N_m = 0.75\ (d/b) + 5.14\ (n+1)$$

gives the best agreement with experimental results, particularly for the case of a thin, stiff ($d/b = 0.5$) top layer. The load-spread technique

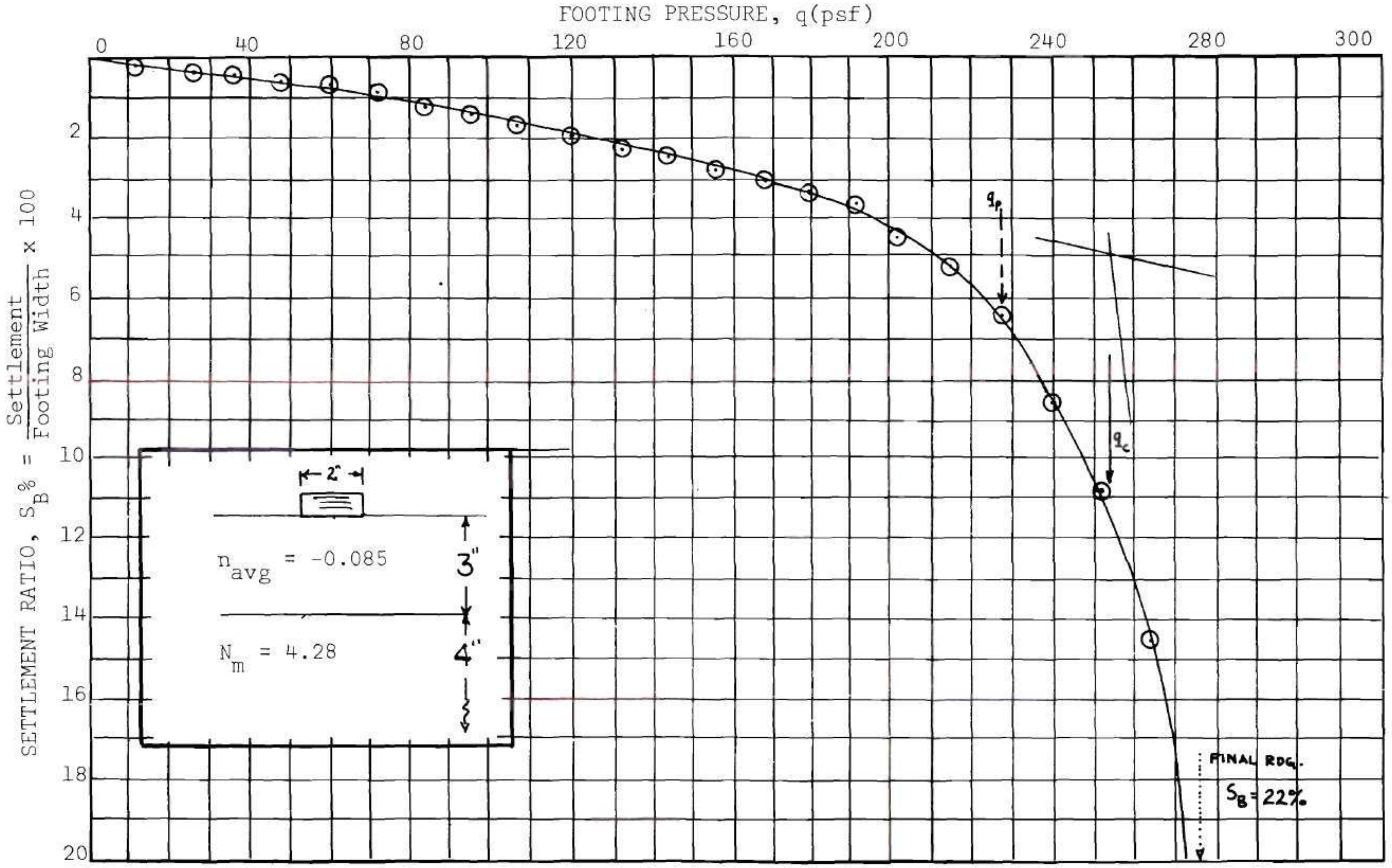

Figure 4-18. Pressure-Settlement Curve: Test 2LS-7, d/b = 3.0, Stiff Layer over Soft Layer Configuration

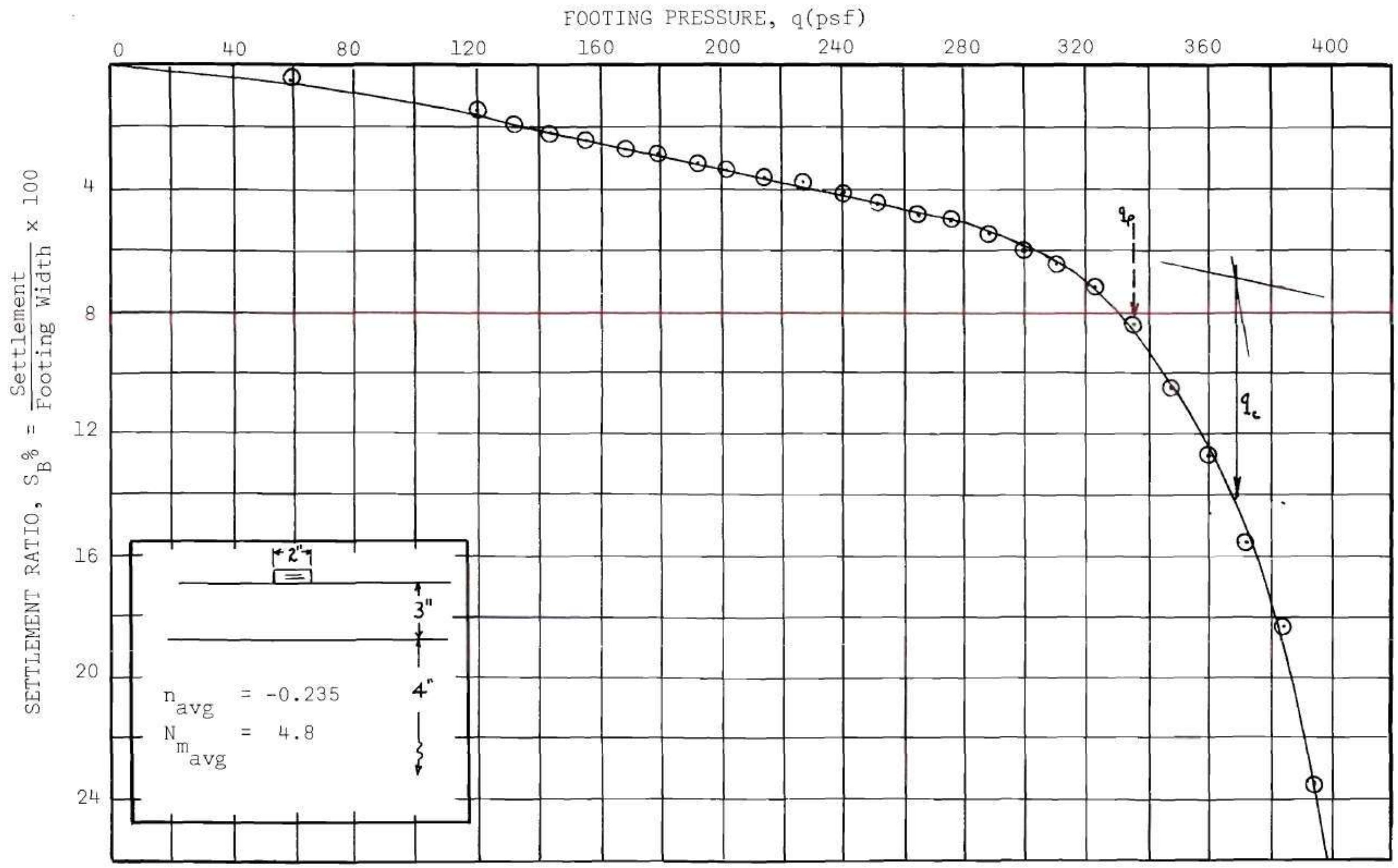

Figure 4-19. Pressure-Settlement Curve: Test 2LS-8, d/b = 3.0, Stiff Layer over Soft Layer Configuration

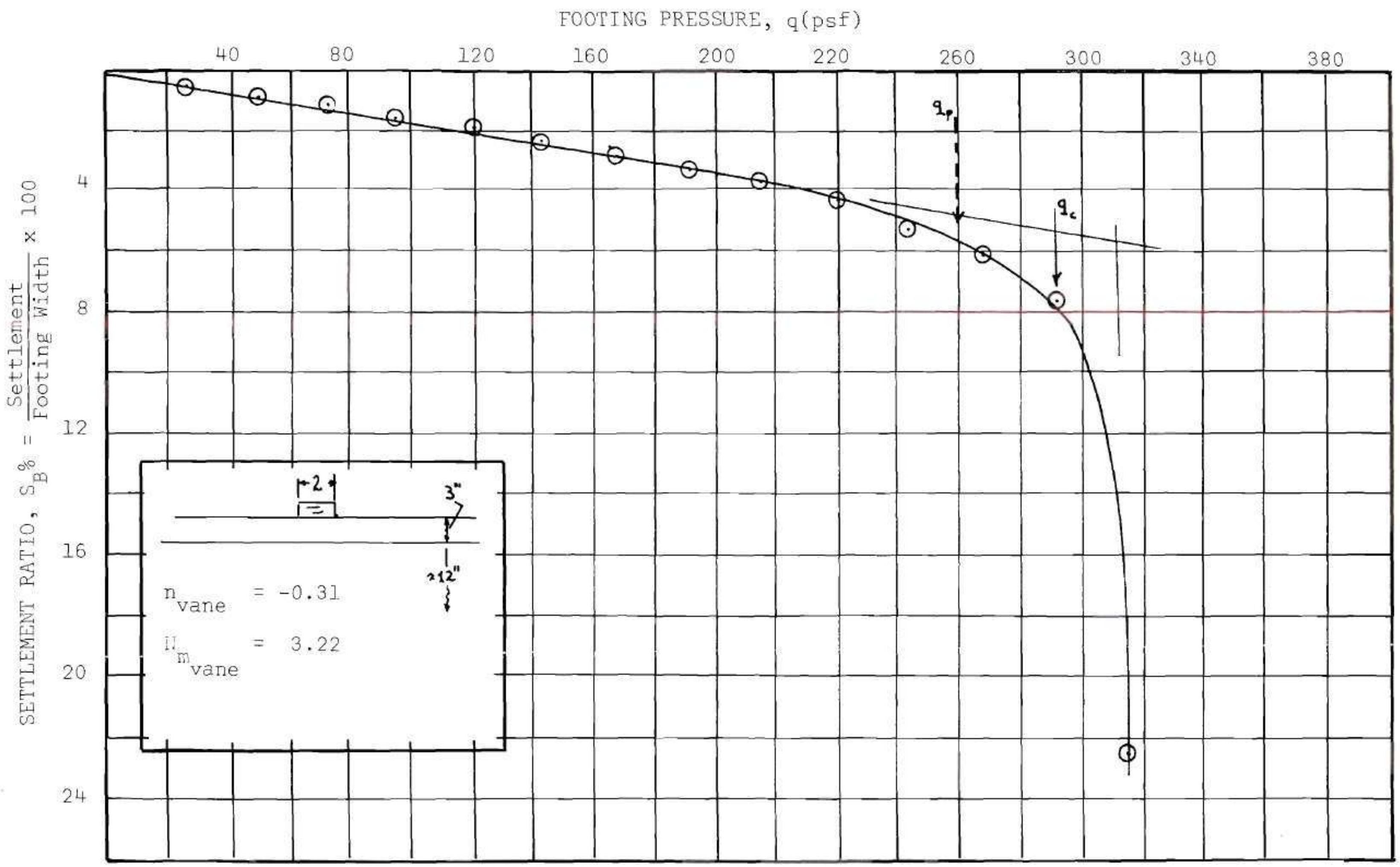

Figure 4-20. Pressure-Settlement Curve: Test 2LB-11, d/b = 3.0
Stiff Layer over Soft Layer Configuration

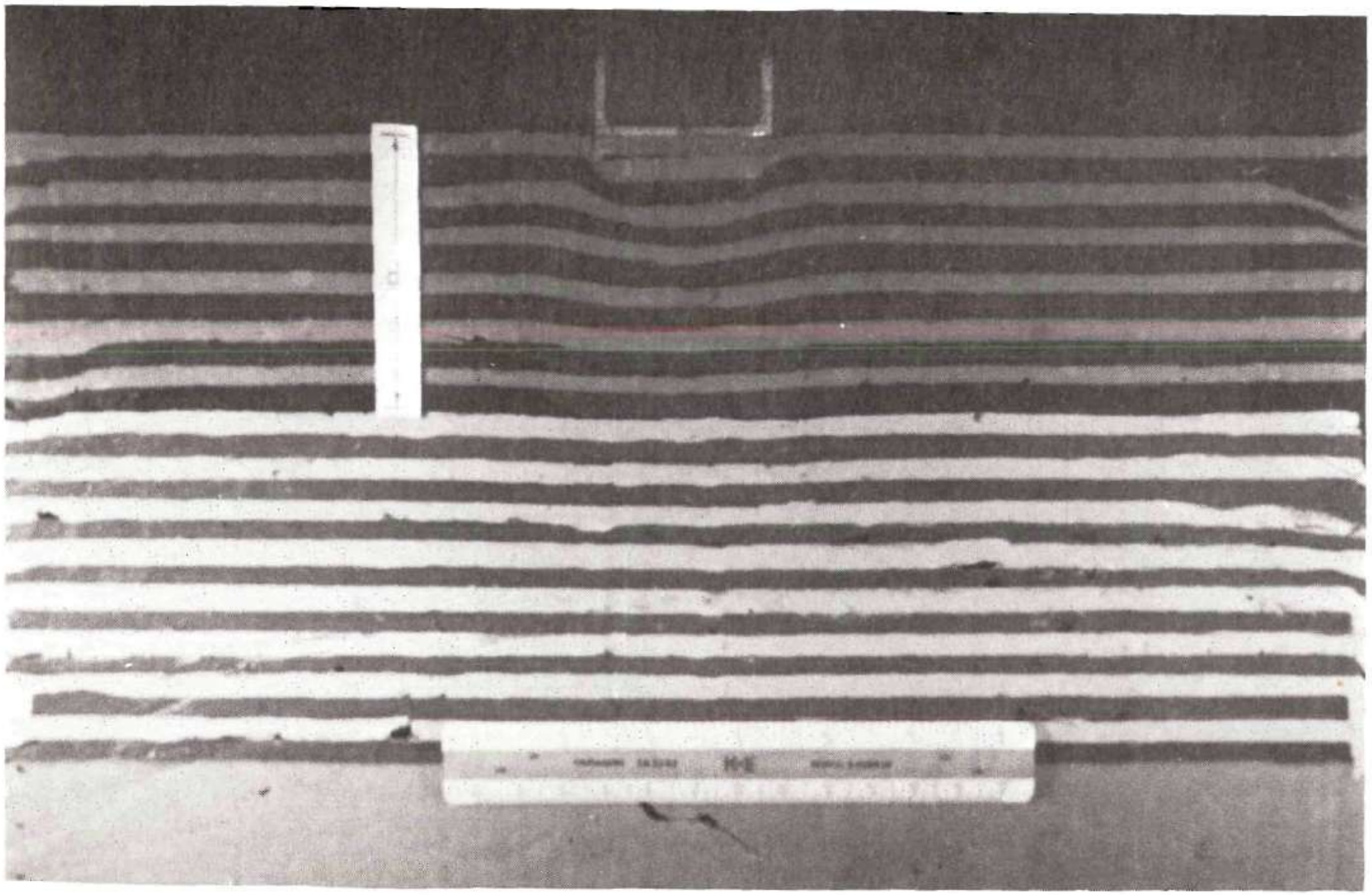

Figure 4-21. Failure Conditions, Test 2LB-11, d/b = 3.0, n = -0.31

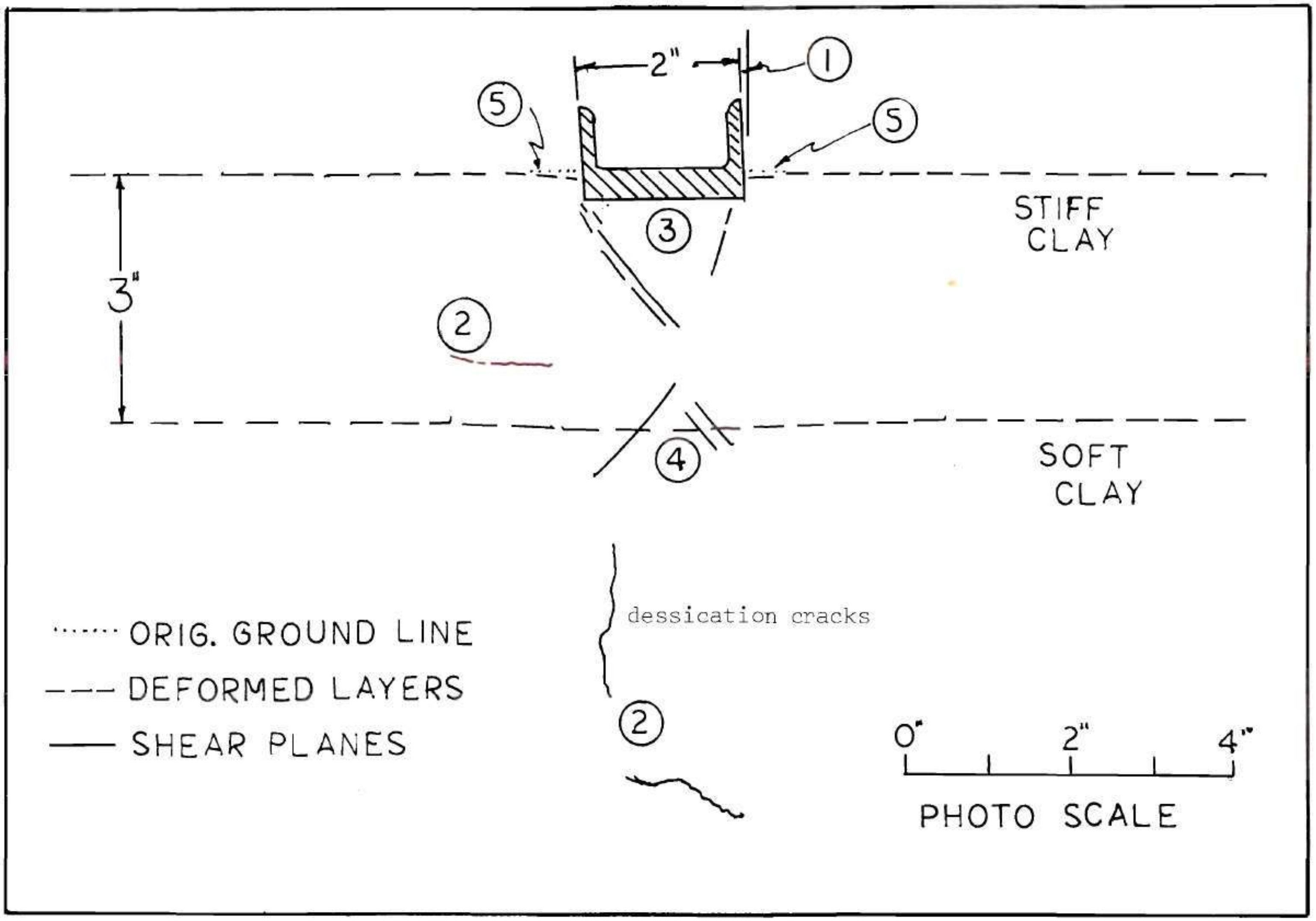

Figure 4-22. Failure Conditions, Test 2LB-11, d/b = 3.0, n ≐ -0.31

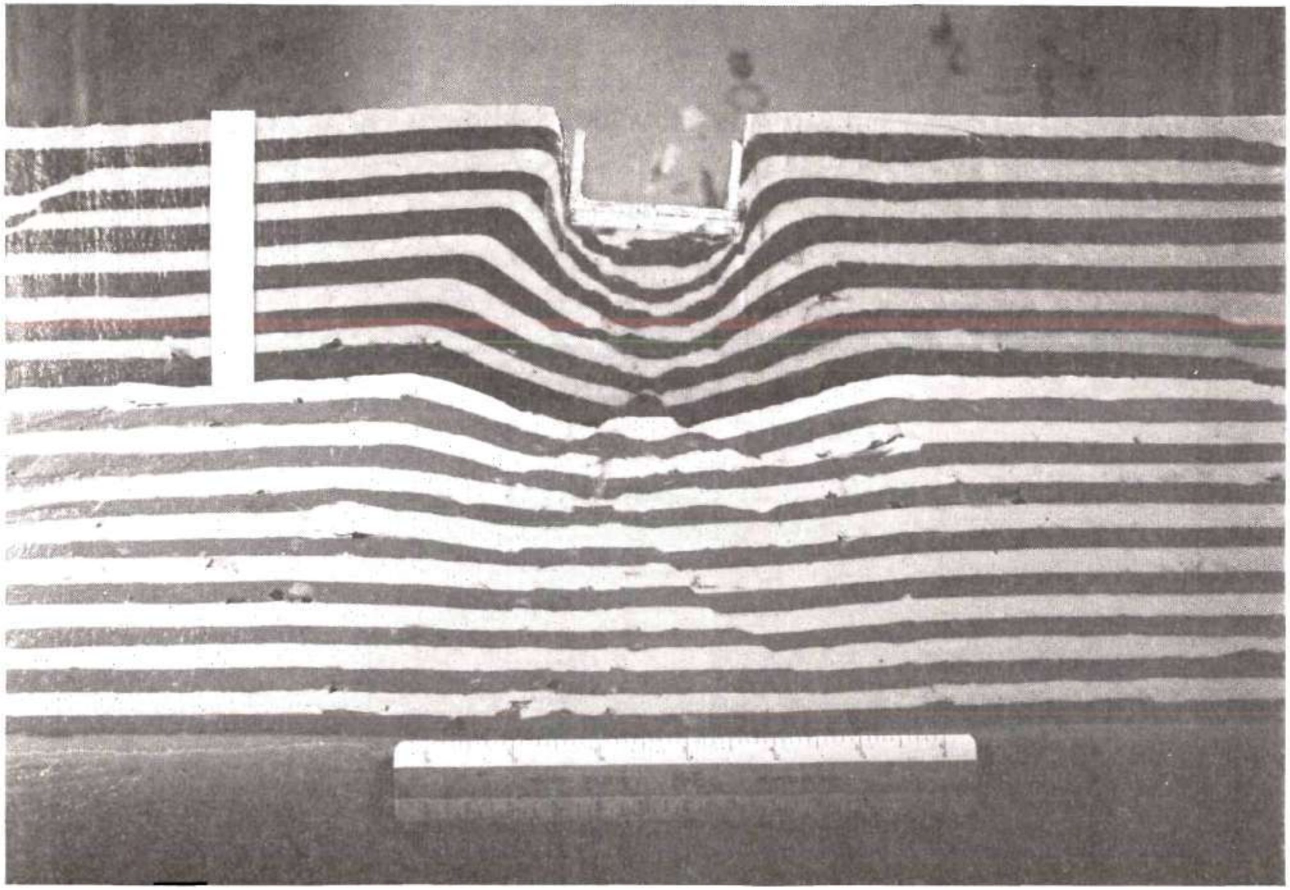

Figure 4-23. Shear Plane Formation at Very Large Strain, Test 2LB-11

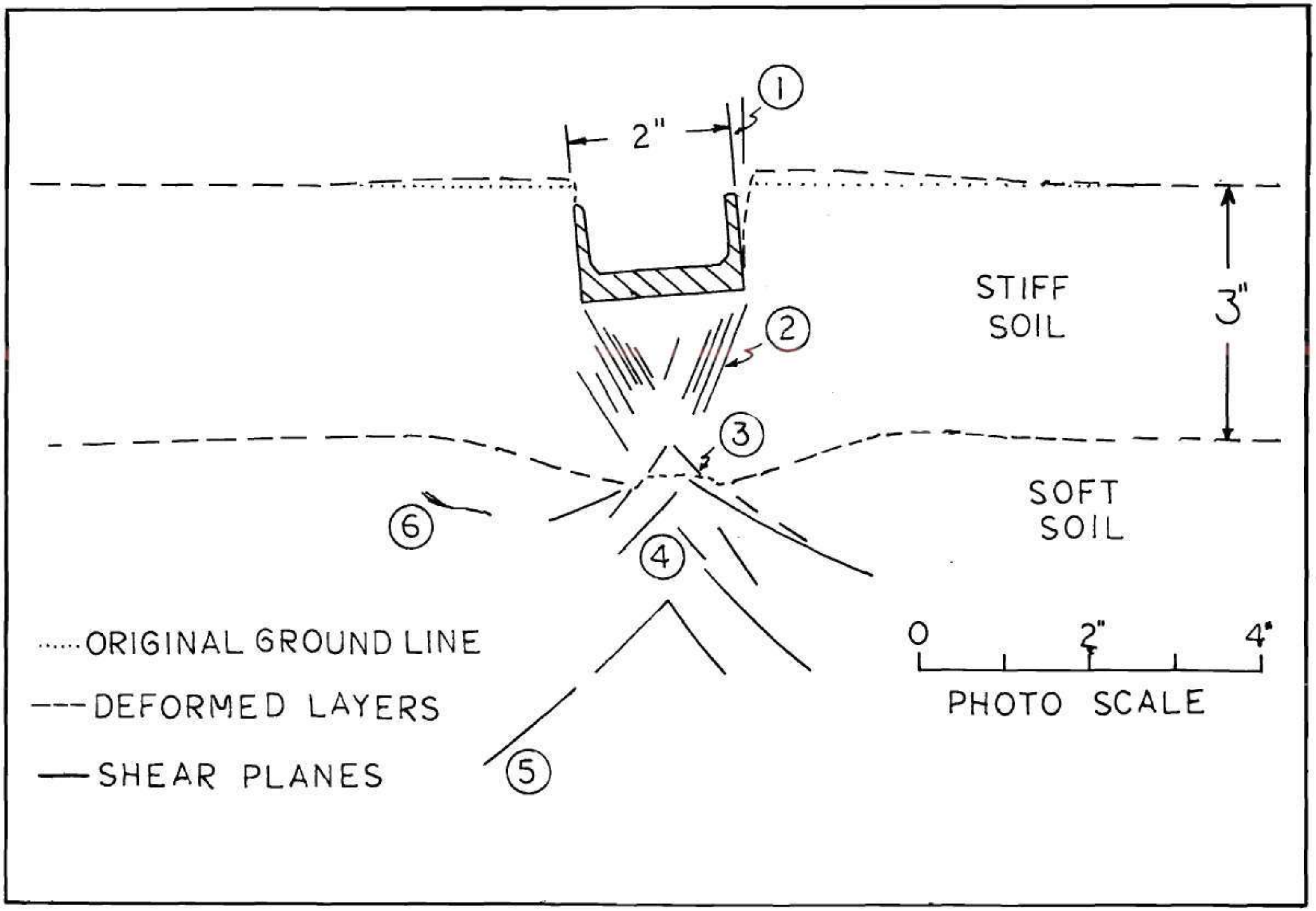

Figure 4-24. Shear Plane Formation at Very Large Strains, Test 2LB-11

of Nixon, although not conservative, gives predictions closer than Button's solution for all $d/b = 0.5$ tests. The other methods noted in the Appendix provide no better solution than Brown's above equation. Although the number of tests performed does not permit a meaningful check of the linear relationships stated in Brown's equation, observations suggest that his assumed failure mode is correct. That is:

3. No vertical punching was noted in the tests but it appears, particularly for the case of $d/b = 2.0$, $n \approx -0.4$ that some form of partial punching does occur.

4. The shear plane formations exhibited in the tests on systems with intermediate top layer thickness are more complex than the theories assume. Most notably, they may be discontinuous. For materials with near coincidental failure strains, but of greatly different strengths (see, for instance, Figure B-2, Appendix B), the paradoxical situation of failure planes initiating in the soft lower layer then propagating upward cannot be totally discounted.

5. For the cases involving a thick stiff top layer ($d/b = 3.0$), the load-spread technique becomes an exceedingly poor approximation. Here Button's and Brown's methods result in overpredictions of about the same magnitude (approximately 30 per cent). It is therefore suggested that Brown's shear punching equation (A.29, Appendix A) be modified for the cases of $d/b \gtrsim 1$ to include the possibility of local shear in the manner used by Terzaghi (1943) for the homogeneous case. The resulting local shear punching solution is then:

$$N_m = 0.75\ (d/b) + 3.4\ (n+1)$$

with the results shown in the table below.

Test	d/b	$N_{m(avg)}$ Experimental	$N_{m(avg)}$ by Above Equation	Per Cent Overprediction
2LS-3	1.0	3.1	2.8	- 9%
2LS-4	1.0	3.0	2.8	- 6
2LS-9	1.0	2.4	2.99	+25
2LS-17	2.0	4.15	3.98	- 4
2LB-19	2.0	2.84	3.77	+25
2LS-7	3.0	4.28	4.80	+11
2LS-8	3.0	5.11	4.85	- 5

It is seen that the reduction gives results which are slightly conservative except in the two cases (2LS-9 and 2LB-19) where data was obtained by vane only.

It should be emphasized that the above reduction is suggested for design purposes in view of the conclusions of points "1" and "4" of the previous discussion. It should also be recognized that the 75 per cent shear punching term of the equation is not necessarily valid for these soil systems, and perhaps this should be reduced as well. However, under the circumstances, it would be blind speculation to make reductions to the coefficient which was empirically derived by Brown.

4.5 Tests Involving Soft Layer over Stiff Layer Configuration

Eight tests were performed in this set. Six involved a d/b of 0.6 and the remaining two were for d/b of 0.8. Referring to the Table of Results, it generally appears that Button's solution gives results which are on the unsafe side. For high n values (as Test 2LS-13) it appears that Tcheng's Prandtl solution is a very poor approximation. Although once again Brown's results yield the closest correspondence, for several cases they too are unconservative (as tests 2LL-20, 2LS-15 and 2LL-23). Since this set of data contains several internal contradictions, it is not possible to describe, with degree of reliability, the magnitudes of overestimation by the various methods.

From Figures 4-29 and 4-30. It is noted that as perimeter shear, 1 , of the soft layer occurs, there is a general thinning out of the layer under the footing, 5 , and a surface bulge, 4 , approximately that of the footing width (as the Meyerhof and Chaplin theory predicts).

Since, once again, the loading for the test of Figures 4-29, 4-30 was somewhat beyond the failure load (Figure 4-28) shear planes into the lower layer (see former figures) are in evidence at 2 . The mark at 3 indicates the lowest level of visible plastic distortion.

4.6 Conclusions and Recommendations: Soft over Stiff Configuration

1. The theoretical solution all result in overpredictions of bearing capacity for the tests performed. Although it is not possible

to set even tentative numerical corrections to the results, it appears that Brown's empirical solution for this case (noted in Appendix A, under the discussion of the Meyerhof and Chaplin theory) could be used as the most reasonable prediction.

2. It appears that the mechanism of failure is primarily an extrusion phenomenon, as suggested by Meyerhof and Chaplin, rather than a shearing of both layers as predicted by the Tcheng-Prandtl and Button solutions.

3. There is a pronounced inflection point (see Figures 4-25, 4-26, points A noted on the pressure settlement curves. It is felt that this is due to a very pronounced non-simultaneous shear mobilization effect which occurs with this configuration. The points A would then correspond to development of shear planes in the lower, stiff layer. However this occurs well beyond what is herein defined as bearing capacity.

4.7 Note on Conclusions; Suggestions for Future Study

As is evidenced by the above, many of the observations are qualitative. In his review of 27 model footing study programs, Roberts (1961) noted the following conclusion:

> Every attempt to use small-scale experiments to verify quantitative relationships or to establish numerical values for bearing capacity led to general frustration. It has been apparent at every turn that a fantastic number of details in experimental technique affect to a high degree the absolute quantitative result.

It can only be added that the difficulties in this regard are greatly compounded when a non-homogeneous soil-system is under test.

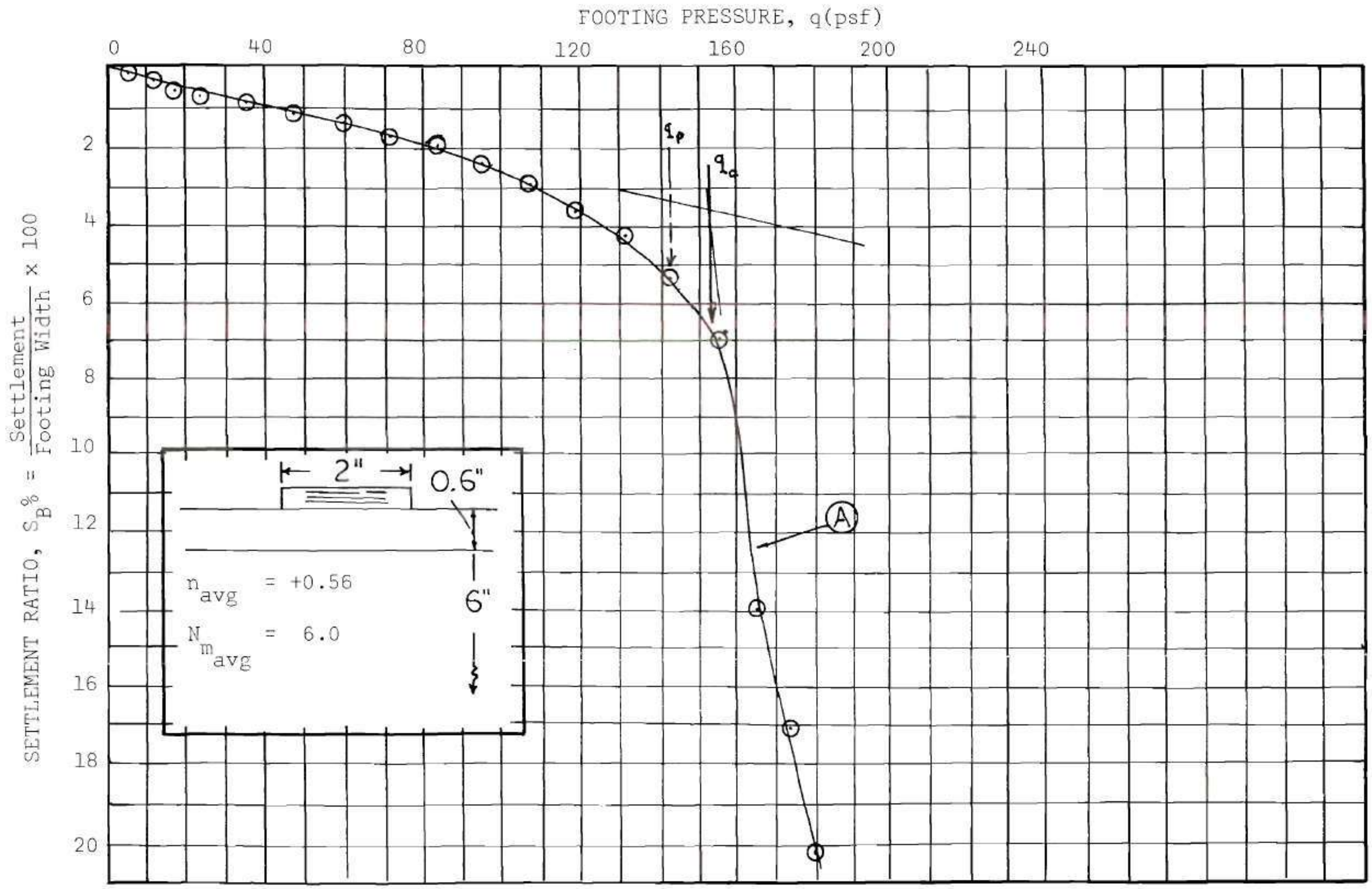

Figure 4-25. Pressure-Settlement Curve, Test 2LS-10, d/b = 0.6, Soft Layer over Stiff Layer Configuration

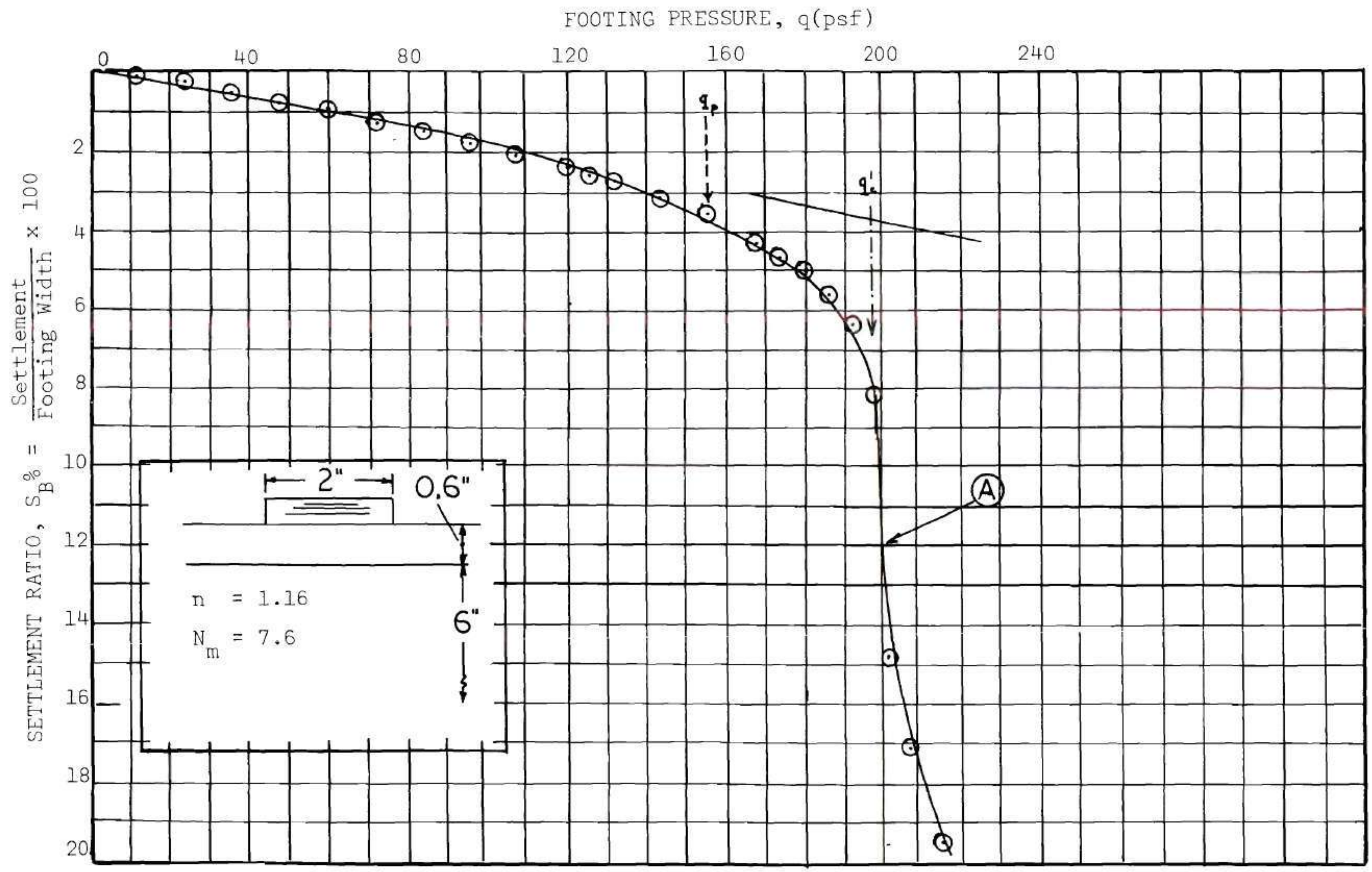

Figure 4-26. Pressure-Settlement Curve: Test 2LS-13, d/b = 0.6, Soft Layer over Stiff Layer Configuration

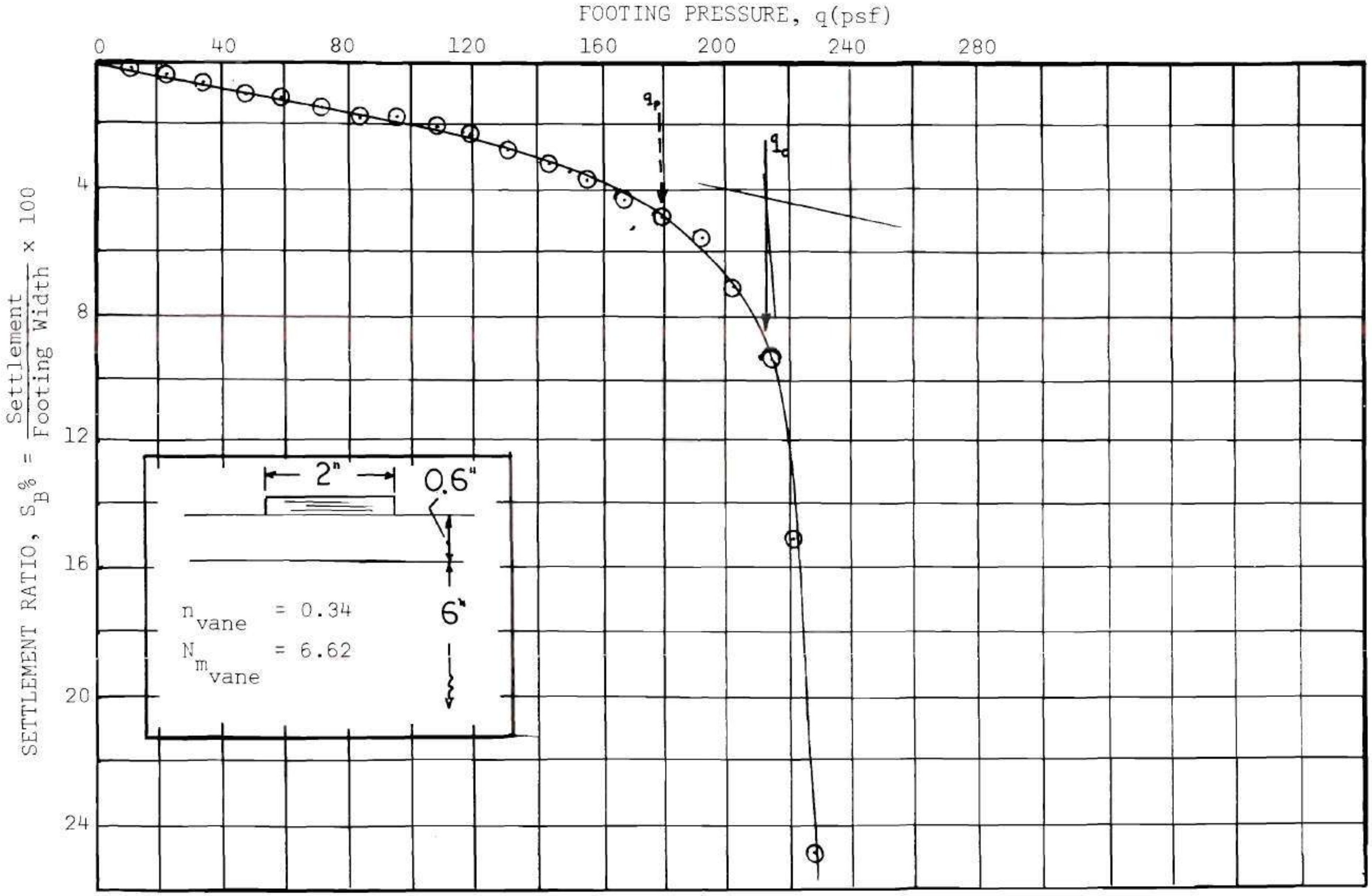

Figure 4-27. Pressure Settlement Curve, Test 2LS-14, d/b = 0.6, Soft Layer over Stiff Layer Configuration

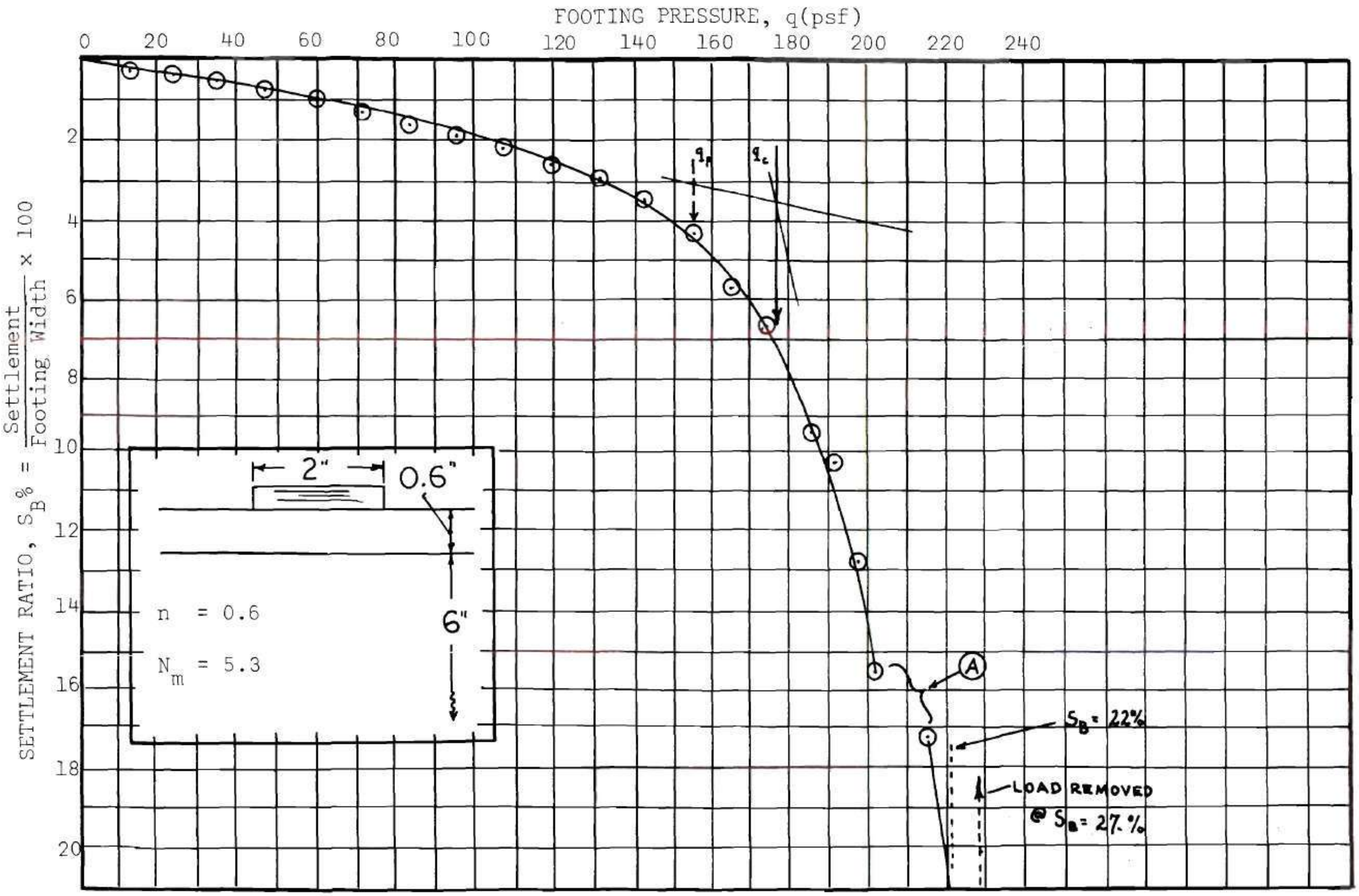

Figure 4-28. Pressure Settlement Curve: Test 2LB-18, d/b = 0.6, Soft Layer over Stiff Layer Configuration

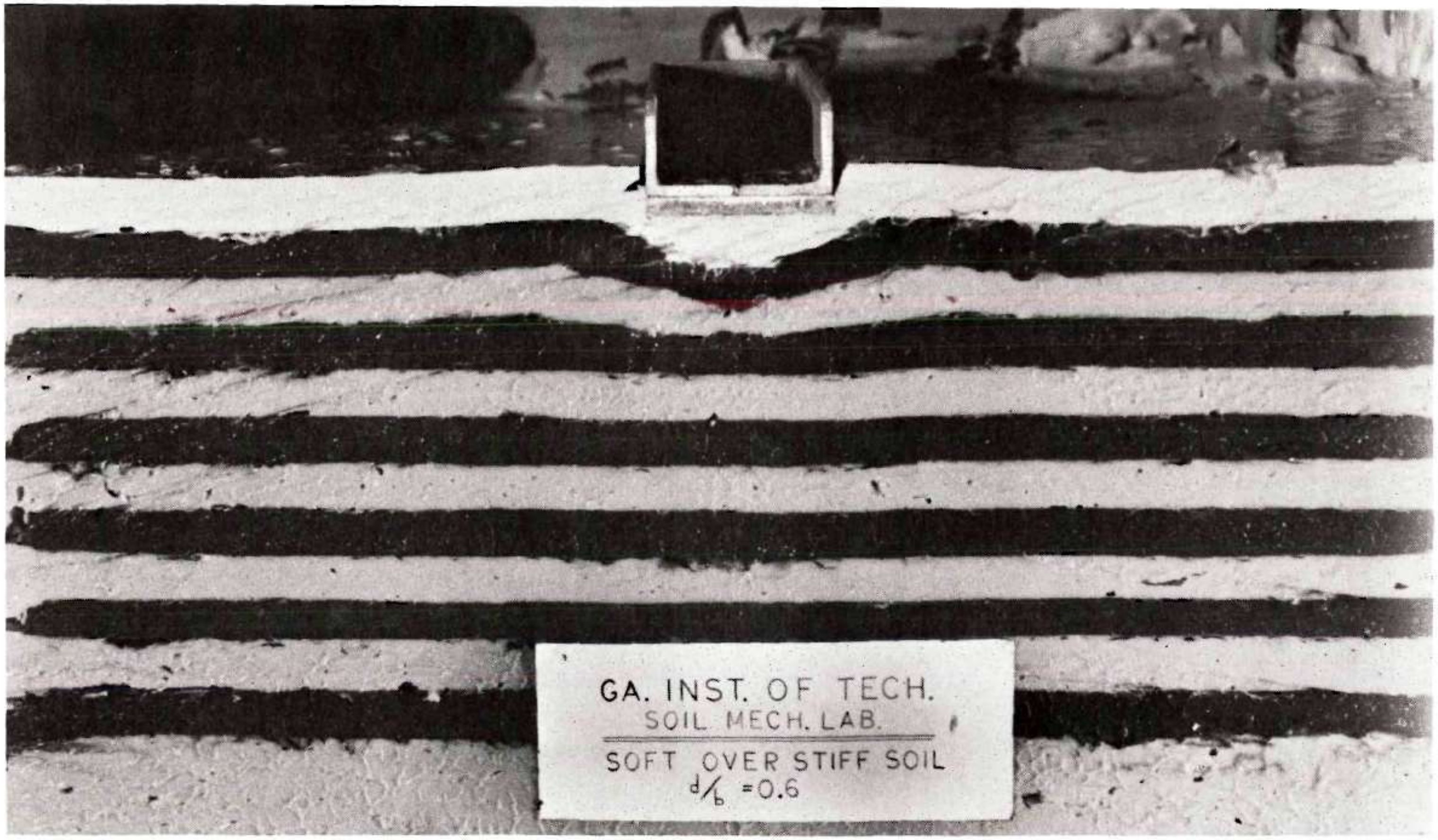

Figure 4-29. Failure Conditions, Test 2LB-18, d/b = 0.6

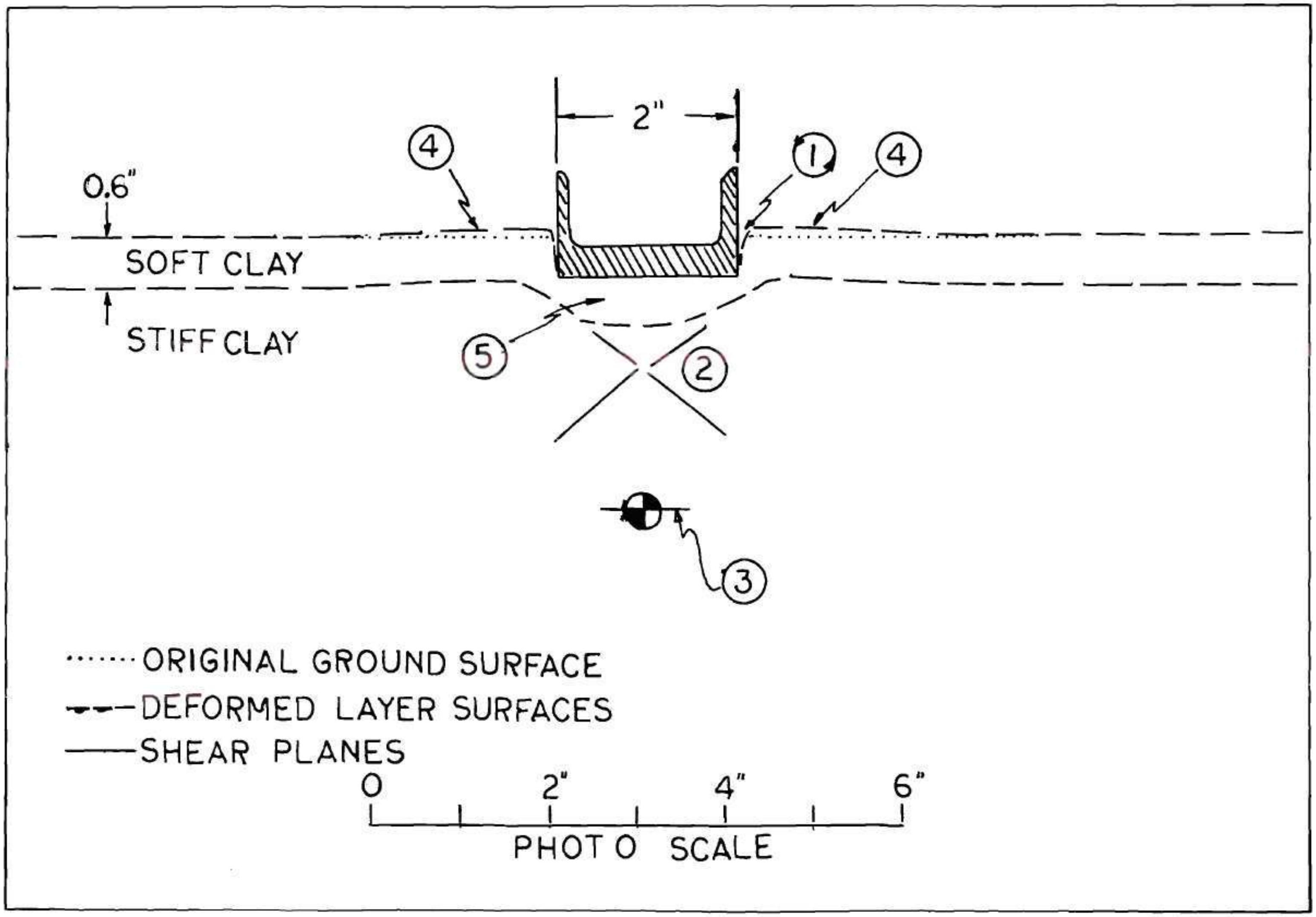

Figure 4-30. Failure Conditions, Test 2LB-18,
d/b = 0.6, n = +0.6

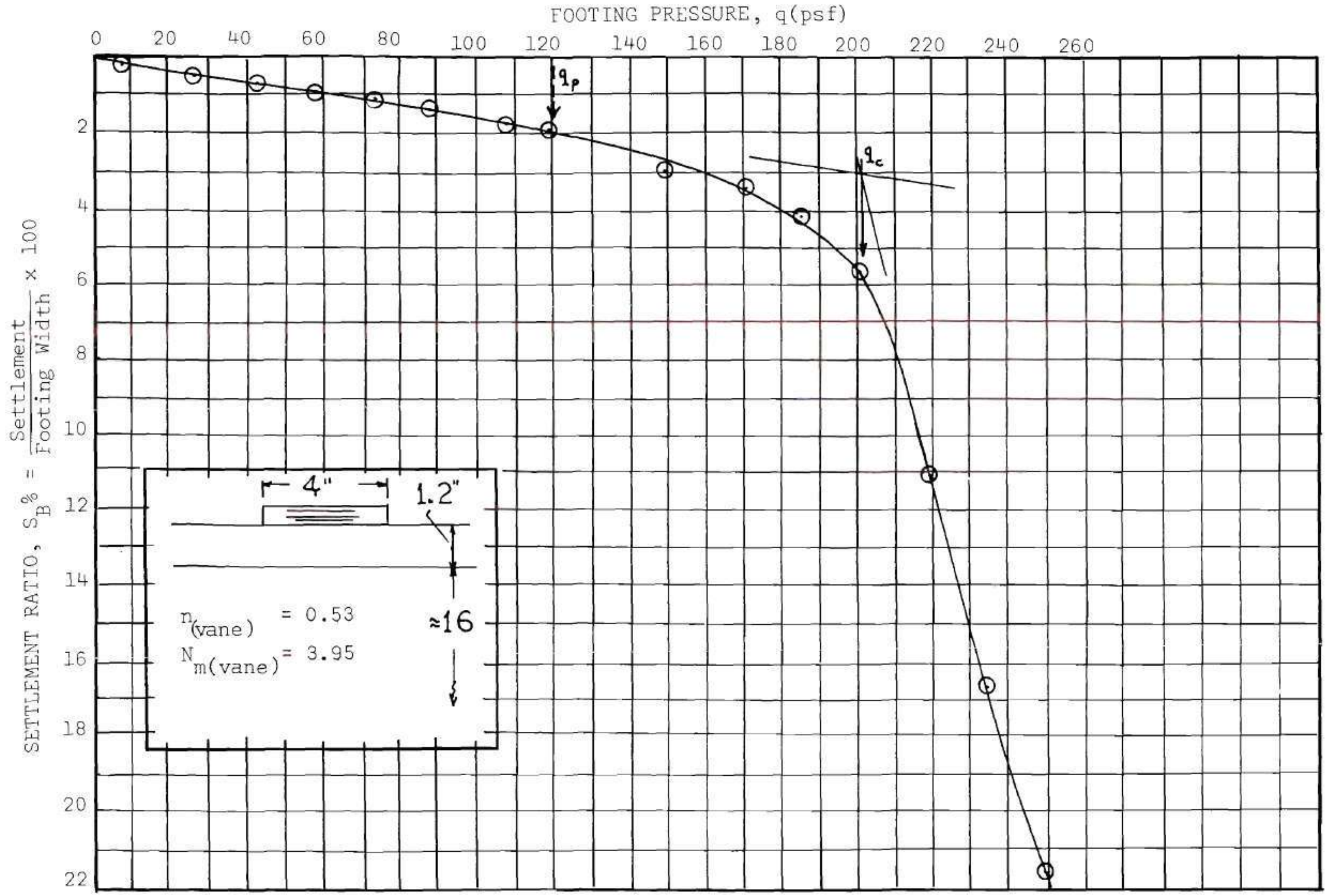

Figure 4-31. Pressure-Settlement Curve: Test 2LL-20, d/b = 0.6, Soft Layer over Stiff Layer Configuration

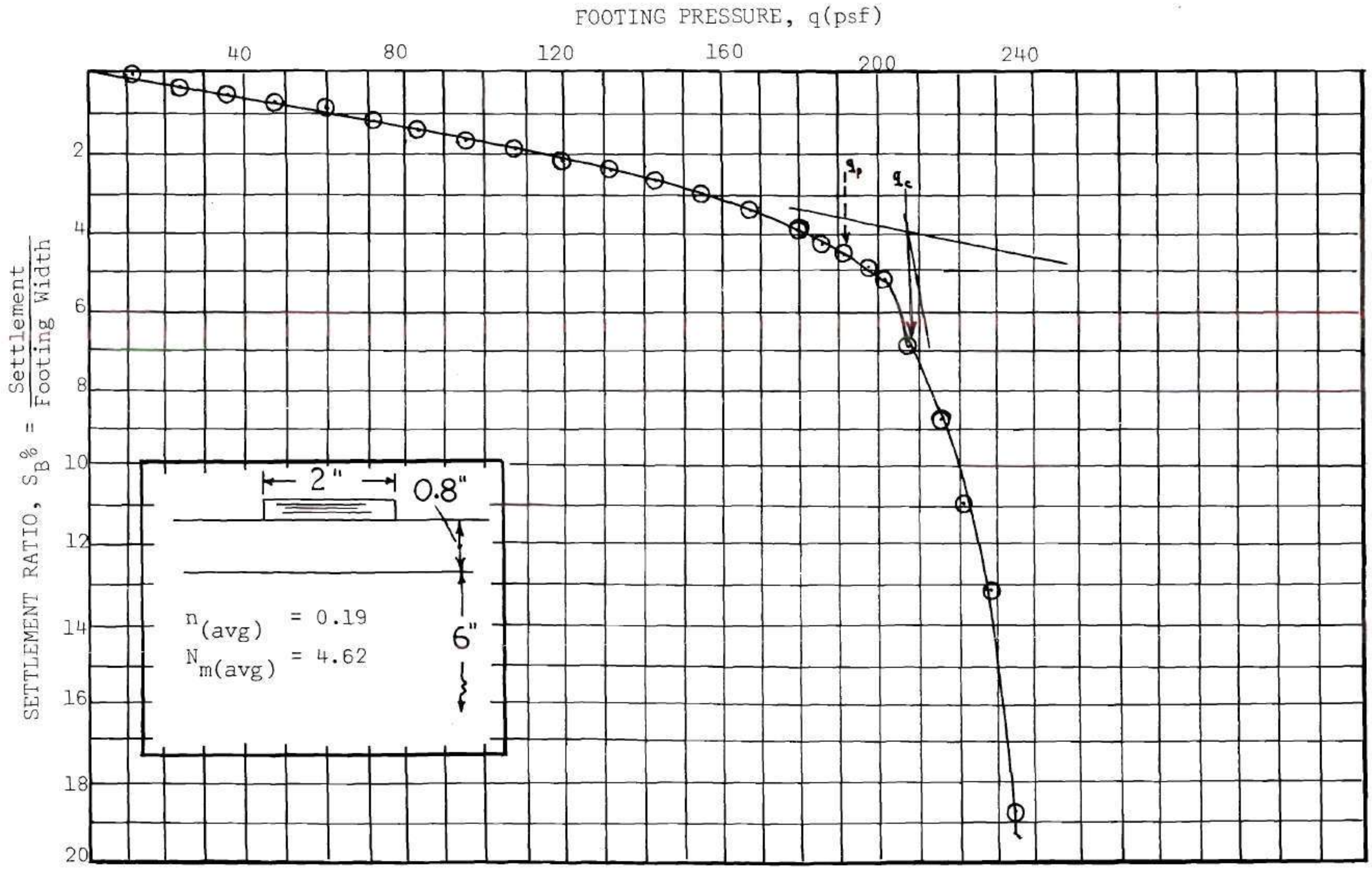

Figure 4-32. Pressure-Settlement Curve: Test 2LS-15, d/b = 0.8, Soft Layer over Stiff Layer Configuration

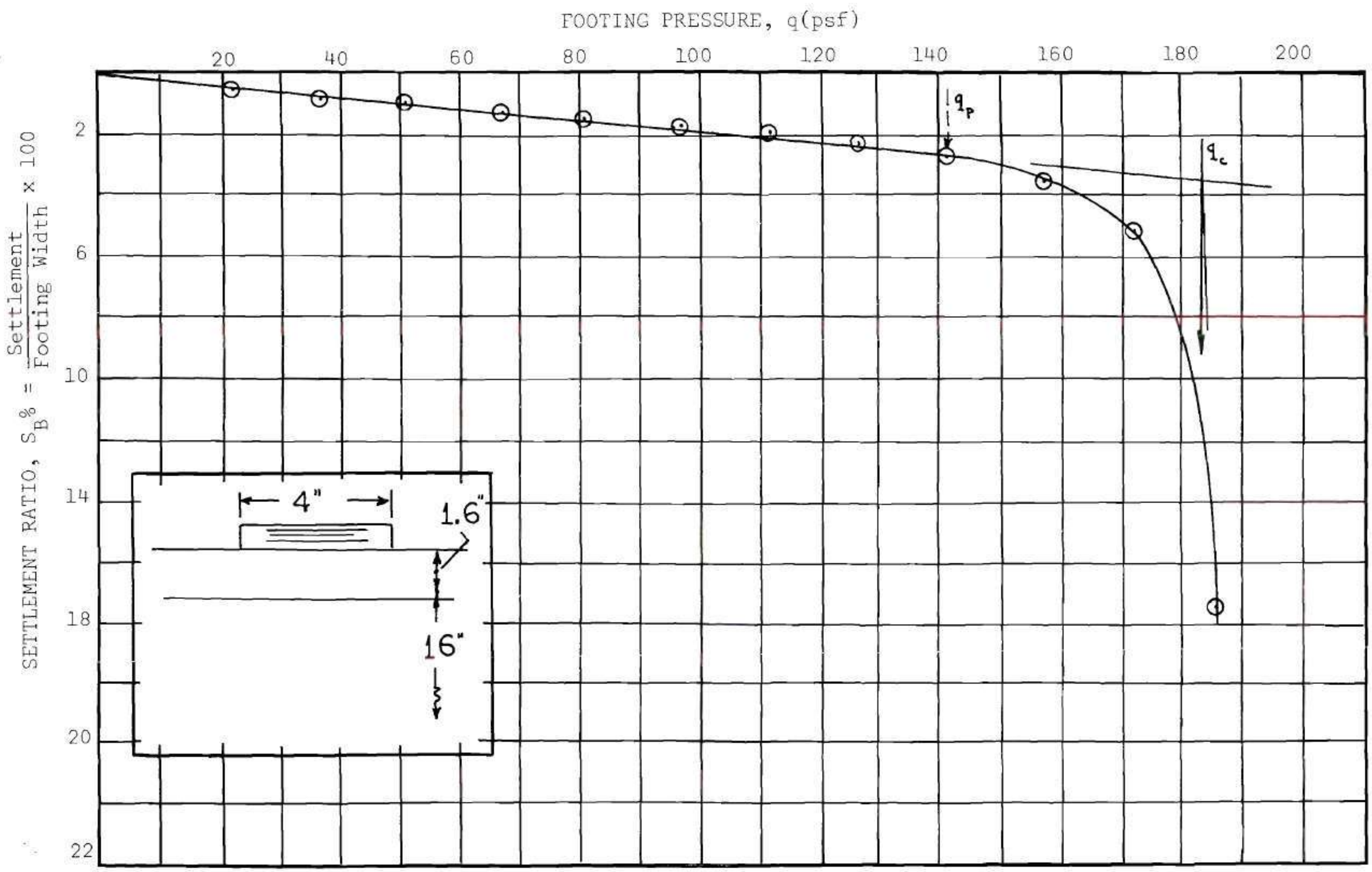

Figure 4-33. Pressure-Settlement Curve: Test 2LL-23, d/b = 0.8, Soft Layer over Stiff Layer Configuration

Future research in this area should involve the use of somewhat stiffer soils, so that accurate strength determinations can be made at the time of testing.

There are some rather obvious extensions of this type of testing. One could use square, circular or rectangular shapes of footings. Flexible strip footings could be made following Tcheng's (1957) innovative designs. A surcharge load could be applied, as in a procedure described by Rocha and Folque (1953).

Finally a full-size test of great practical interest could be done in the lab by using a replica of a grouserplate, i.e., the treadplate element of a tracked vehicle, instead of a model foundation footing. Theoretical solutions are available for the grouserplate bearing capacity problem (Haythornthwaite, 1961), but its assumed failure mechanisms have not been verified by test.

APPENDIX A

THE EXISTING METHODS FOR ESTIMATING THE ULTIMATE BEARING CAPACITY OF STRIP FOOTINGS FOUNDED ON LAYERED COHESIVE SOILS

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THE EXISTING METHODS FOR ESTIMATING THE ULTIMATE BEARING CAPACITY OF STRIP FOOTINGS FOUNDED ON LAYERED COHESIVE SOILS

1. Methods Based on Average Soil Strength Parameters

The most commonly employed method of estimating the bearing capacity of a layered foundation soil is to base the computations on an "equivalent" homogeneous soil, having a strength which is the average value over the anticipated shear surfaces. Typical of this approach is that advanced by Skempton (1951) who proposes,

> If the shear strength within a depth of approximately two-thirds times the footing width beneath the foundation level does not vary by more than about ±50 per cent of the average strength in that depth, then this average value of shear strength may be used in (the general bearing capacity) equation.

2. Methods Based on an Assumed Load Spread

These methods have been developed for the case of stiffer soil overlying deep softer deposits. They are considered as conservative, approximate solutions, in which one either:

A. Ignores the strength contribution of the top stiff layer, assuming bearing capacity is developed only in the lower layer, or,

B. Modifies the above solution to include the shear strength mobilized in the top layer.

The rationale for such methods is the intuitive argument that the stiff top material acts as a "natural raft" which causes a reduction in vertical stress on the soft soil below.

2A. Solution "A"

The former approach enjoys the greater popularity in the literature. Since the strength of the top layer is ignored, the problem is merely one of determining the bearing capacity of the homogeneous soft subsoil, and then increasing this value by some function of the depth of stiff layer to footing width ratio, f(d/B), defining the load spread benefit. The mathematical expression is then:

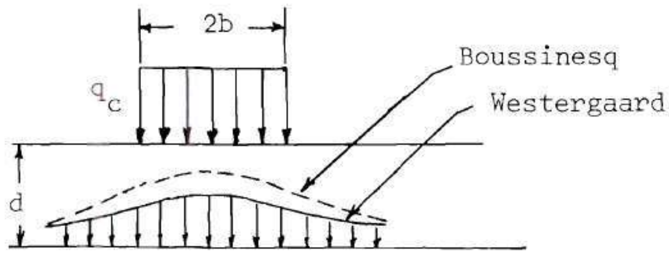
$$q_c = f(d/B)\ c_2 N_{c_2} \qquad (A.1)$$

where

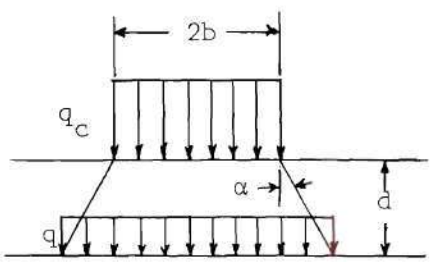
q_c = ultimate bearing capacity at the footing level, and

$c_2 N_{c_2}$ = bearing capacity of the softer soil

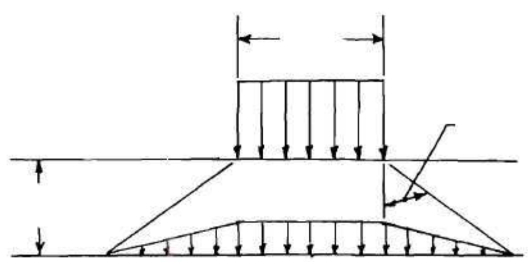
The term f(d/B) will, of course, depend on the load spread assumed. Those "stress roofs" commonly used are illustrated in Figure (A-1). An accurate determination of the true stress distribution seems unwarranted (Taylor, 1948). The range of possible spreads is covered by the extremes of the Nixon and the Koegler methods, with the Boston Code providing mean values throughout most of the d/B range.



a. Elastic Methods
(Taylor, 1948;
Terzaghi and Peck, 1968)



b. Uniform Stress Reduction
Boston Code, $\alpha = 30°$
(Taylor, 1948)
Shellhaven Tanks, $\alpha = 22\frac{1}{2}°$
(Nixon, 1949)



c. Koegler Method
(Taylor, 1948)

Figure (A-1). Assumed Vertical Stress Distributions on the Surface of a Buried Stratum

Referring to Figure (A-1), for a strip footing the Boston Code yields:

$$q/q_c = \frac{B/d}{B/d + 2 \tan 30°} \tag{A.2}$$

where

q = pressure at the surface of the buried stratum, and

q_c = pressure at the level of the footing base

Taylor (1948) presents solutions in graphical form, using a plot of q/q_c against B/d. It is desirable, however, to express these solutions in terms of the modified bearing capacity equation, a form first proposed by Button (1953). For layered clays in the $\phi = 0$ analysis, this equation is:

$$q_c = c_1 N_m \tag{A.3}$$

where

q_c = ultimate bearing capacity of the two-layer system

c_1 = undrained shear strength of the top layer, and

N_m = modified bearing capacity factor

Substituting Equations (A.3) and (A.2) into (A.1) and solving for N_m:

$$N_m = \frac{\frac{c_2}{c_1} N_{c_2}}{\frac{B/d}{B/d + 2 \tan 30^\circ}}$$

Transforming the equation into Button's (1953) notation for the sake of consistency, and recalling that $B = 2b$, $n = (c_2/c_1) - 1$, and $N_{c_2} = 5.14$, one obtains:

$$N_m = [(n+1)\ (5.14 + \rho(d/b))] \qquad (A.4)$$

where

$\rho = 2.97$ for the Boston Code stress distribution.

Similarly the Nixon and (based on the maximum stress roof value q_k) the Koegler methods can be represented by Equation (A.4). For Nixon's method $\rho = 2.13$; and for Koegler's method $\rho = 3.68$.

2B. Yamaguchi's Method

The conventional load spread technique has been modified by Yamaguchi (1963) to include the strength contribution of the top layer. Although he limited his analysis to the case of a sand layer overlying soft clay, the method can also be applied to a two-layer clay soil.

Yamaguchi chose a 1:2 ($\rho = 2.57$) load spread (see Figure A-2) and took the assumed top layer strength contribution along a vertical plane as a uniform equivalent surcharge imposed on the Prandtl figure.

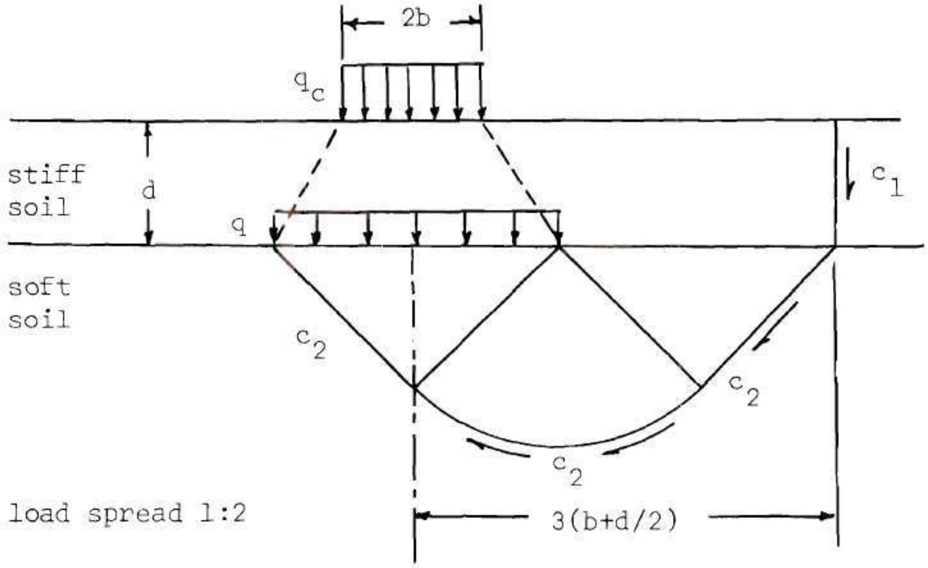


Figure (A-2). Yamaguchi's Method for Determining the Bearing Capacity of a Two-Layer Subsoil

From Figure (A-2),

$$q/q_c = \frac{1}{1 + (d/2b)} \tag{A.5}$$

but

$$q_c = f(d/b)[c_2 N_{c_2} + \frac{c_{1d}}{3(b + d/2)}]$$

or

$$q_c = (1 + d/2b)[c_2 N_{c_2} + \frac{c_{1d}}{2(b + d/2)}] \tag{A.6}$$

then substituting:

$$q_c = c_1 N_m \tag{A.3}$$

into Equation (A.6) and again using

$$n = (c_2/c_1) - 1$$

one obtains:

$$N_m = \left(1 + \frac{d}{2b}\right) \left((n + 1)N_{c_2}\right) + \frac{d}{3b} \tag{A.7}$$

where

$$N_{c_2} = 5.14$$

3. Methods Based on Bearing Capacity Solutions for Homogeneous Soils

Two methods which were developed for homogeneous soils have been extended to the layered soil problem:

A. Fellenius' solution (Button, 1953).

B. Prandtl's solution (Tcheng, 1957; Brown, 1967).

3A. Button's Method

This method uses the Fellenius solution to determine the bearing capacity of strip footings founded on two-layer clays. It is the condition of a subsoil with uniform, different layer strengths which will be considered here. It should be pointed out, however, that Button's

(1953) paper also presented solutions for the case of a linear strength variation through the top layer.

Srinivasan and Siva Reddy (1967) extended Button's method to include strength anisotropy, while Raymond (1967) considered the case of a linear variation of strength with depth for the lower layer. The latter uses a hybrid solution: the Nixon load spread (Figure A-1) and a modified Fellenius analysis of the lower layer.

Since such refinements as the above were not considered in the present research, "Button's method" is herein defined as the case shown in Figure (A-3).

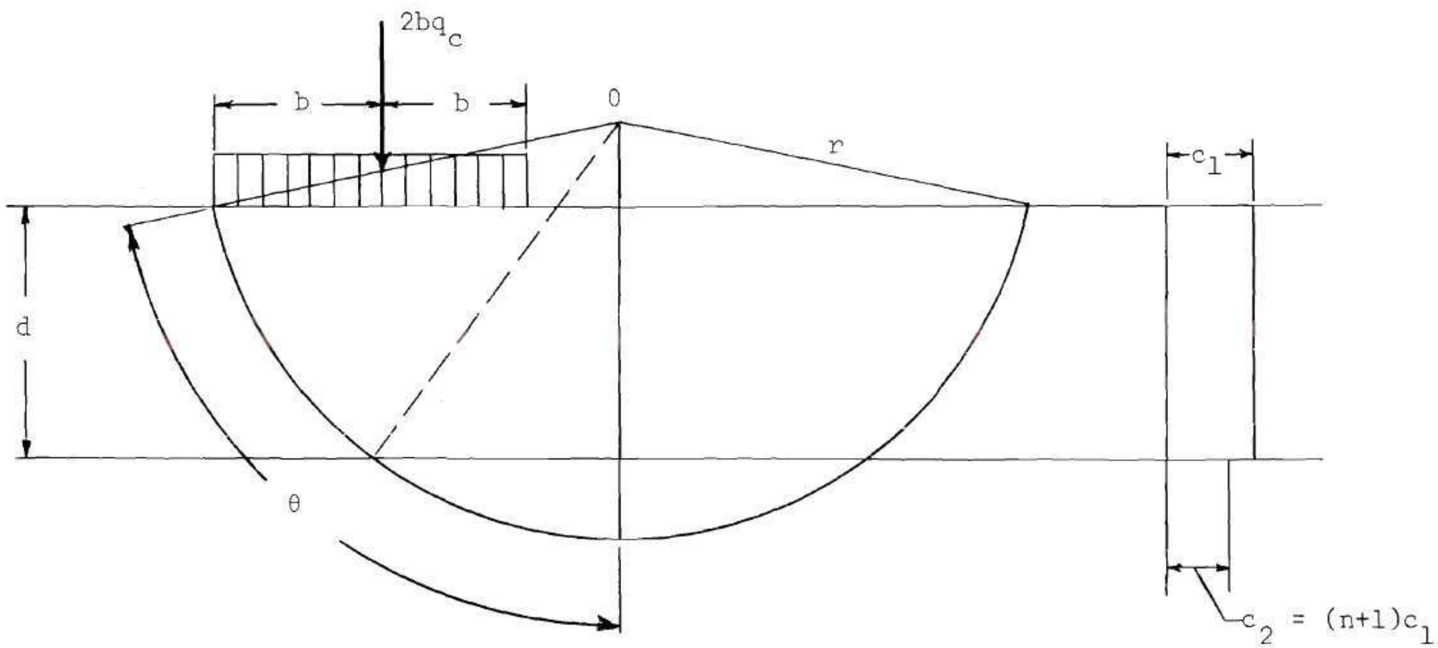
Considering a slice of unit thickness into the plane of the page, at impending failure, the driving moment about point 0 equals the resisting moment mobilized along the cylindrical slip surface, or,

$$2q_c b(r \sin \theta - b) = 2r^2\theta\, c_1 + 2r^2\, nc \cdot \cos^{-1}(\cos\theta + d/r) \quad \text{(A.8)}$$

Solving for $N_m = q_c/c_1$ and substituting non-dimensionalized parameters $r' = r/b$ and $d' = d/b$, one obtains

$$N_m = \frac{r'^2(\theta + n \cos^{-1}[\cos \theta + d'/r'])}{r'(\sin\theta) - 1}$$

The above equation cannot be entered directly for a solution since there is an additional restriction on the values of θ and r'. The point 0 (Figure A-3) corresponds to a critical circle yielding the minimum value of q_c.



(a) Failure Surface Geometry

(b) Assumed Shear Stress Distribution Along Slip Surface

Figure (A-3). Button's Solution for the Ultimate Bearing Capacity of a Two-Layer Clay: Component Layers of Uniform Strength

The d and b are fixed values, while θ and r' are variables. The critical slip circle must satisfy

$$\frac{\partial Nm}{\partial \theta} = \frac{\partial Mm}{\partial r'} = 0 \qquad (A.10)$$

Button reduced the various solutions to a series of curves (Figure A-4) which were obtained by the following procedure.

He chose a value of n and d', thus setting the conditions of geometry and strength for the foundation system. N_m values were calculated by Equation (A-9) for various trial r' and θ values.

Button then plotted N_m vs. θ for fixed r' values and applied the restriction (Equation A.10) $\frac{\partial Nm}{\partial \theta} = 0$ to the series of curves, finding values of r' and θ which are partial solutions.

Similarly a plot of N_m vs. r' for fixed values of θ with the condition (Equation A.10) $\frac{\partial Nm}{\partial r'} = 0$ yielding other values of r' and θ which are also partial solutions.

The simultaneous condition

$$\frac{\partial Nm}{\partial \theta} = \frac{\partial Nm}{\partial r'} = 0$$

was found by a plot of the above two sets of partial solutions on a θ vs. r' graph. Their intersection point gave the critical r' and θ for the circle at center O. Substituting these critical values into Equation (A.9) yielded N_m for the particular d/b and n chosen. As these were varied, the curves of Figure (A.4) were generated.

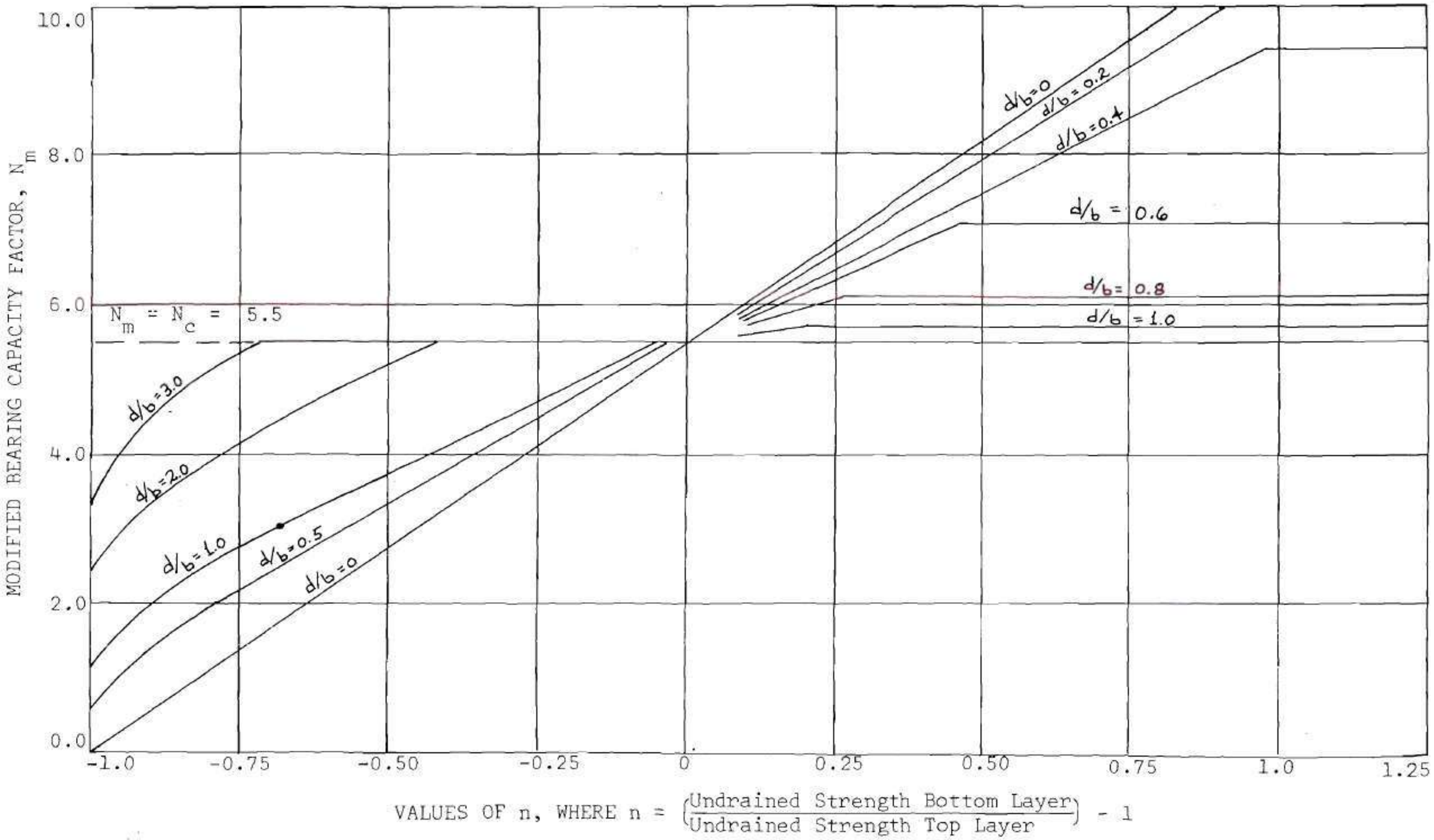


Figure (A-4). Button's Solution for the Bearing Capacity of a Two-Layer Clay: Component Layers of Uniform Strength

3B.1. Tcheng's Prandtl Solution

Tcheng was the first investigator to present model footing test results (in the usual soil mechanics literature) for the layered soil bearing capacity problem. The 1957 reference cited, unfortunately, presents results only for the case of a granular layer overlying a cohesive one. However, this 1957 reference, in turn, cites a work which is highly germane to the present research, i.e., Tcheng's doctoral thesis.* Regrettably, the work can only be indirectly referenced herein; the following treatment, it should be emphasized, is not based on the original work, but rather on a discussion of the methods as given by Brown (1967).

The Prandtl (1920) bearing capacity theory for homogeneous soil was modified by Tcheng (1956) to provide a solution for the two-layer clay case. Depending upon the d/b ratio, Tcheng's Prandtl solution leads to two different analytical expressions.

The first case is for $d/b \leq 1$, in which the following obtains: (see Figure A-5 on page 98).

The tangential component of resistance moment about point $0 = \Sigma M_o \tan =$

$$4c_1 bd + 4c_2 b(b-d) + \pi c_2 b^2$$

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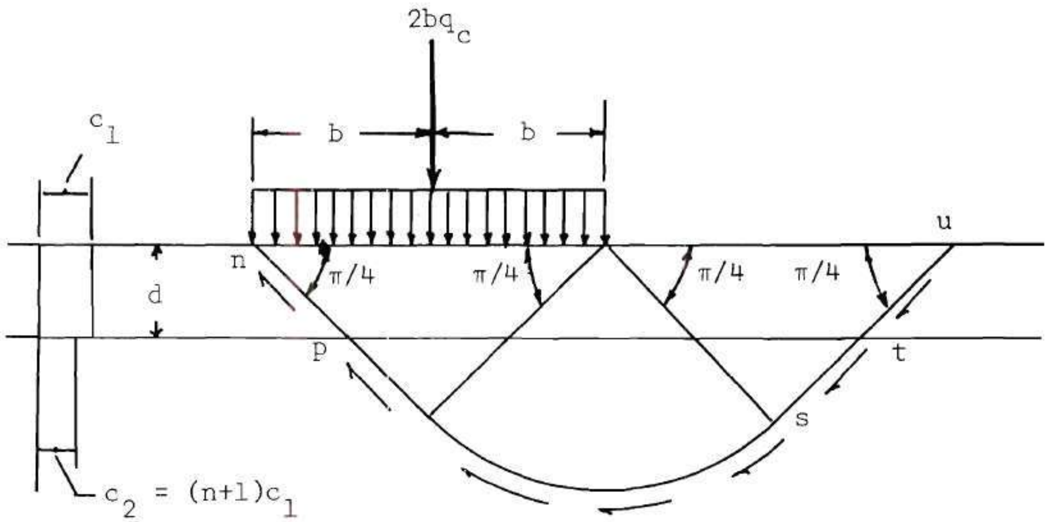
*Tcheng, Yuan (1956), "Pouvoir Portant d'un Solide Composé de Deux Couches Plastiques Différentes," Ph.D. diss., University of Paris.



Assumed Shear Stress Distribution Along Slip Surfaces in Tcheng's Solutions

NOTE: Analysis Assumes that Principal Planes Within Wedge One Remain Vertical (σ_3) and Horizontal (σ_1); It Is a "Dead" Elastic Wedge

Figure A-5. Tcheng's Prandtl Solution for the Bearing Capacity of a Two-Layer Clay: Case Where $d/b \leq 1$ (after Brown, 1967)

The normal component of resistance moment about point

$$0 = \Sigma M_{o/norm} = (q_c - c_1)(d\sqrt{2})(b\sqrt{2} - \frac{d\sqrt{2}}{2} + \frac{1}{2}(q_c - c_2) \cdot$$

$$(b\sqrt{2} - d\sqrt{2})^2 - \frac{1}{2} c_2(b\sqrt{2} - d\sqrt{2})^2 -$$

$$c_1\sqrt{2}d\ (\sqrt{2}b - \sqrt{2}d)$$

At limiting equilibrium $\Sigma M_{o_{tan}} + \Sigma M_{o_{norm}} = 2q_cb^2$; simplifying and solving for q_c yields:

$$q_c = (c_1-c_2)\,\frac{2d^2}{b^2} + (2+\pi)c_2 \tag{A.11}$$

now recall $N_m = q_c/c_1$ and $n = (c_2/c_1) - 1$, then, Equation (A.11) provides

$$N_m = 5.14(n+1) - \frac{2d^2}{b^2}\,n \tag{A.12}$$

The case where $\frac{d}{b} \geq 1$ is similarly solved: (Figure A-6)

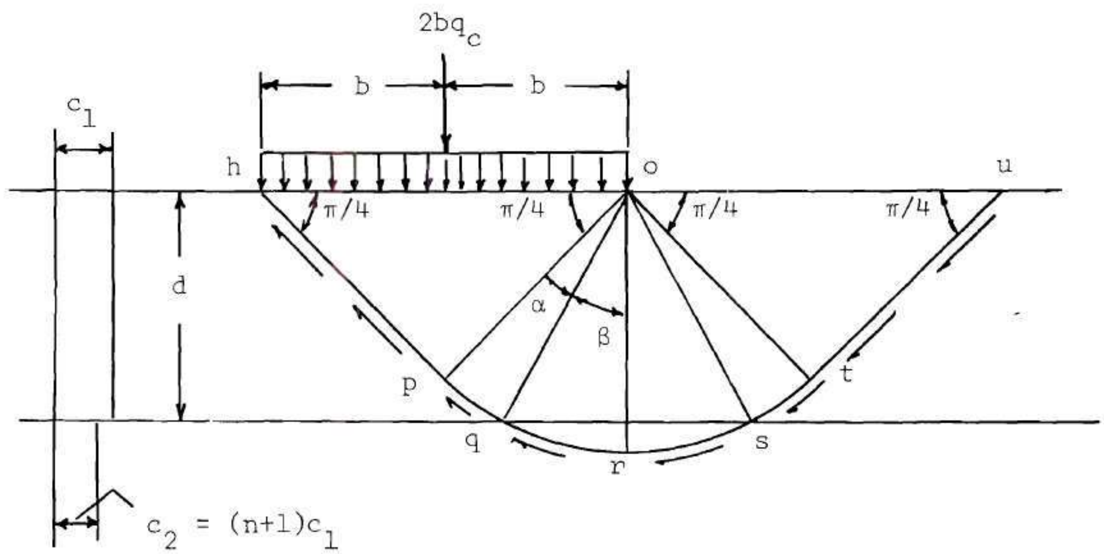


Figure A-6. Tcheng's Prandtl Solution for the Bearing Capacity of a Two-Layer Clay: Case Where $d/b \geq 1$ (After Brown, 1967)

$$\Sigma M_{o_{tan}} = 4b^2(c_1 + \alpha c_1 + \beta c_2)$$

$$\Sigma M_{o_{norm}} = (q_c - c_1)b^2 - c_1 b^2$$

for equilibrium

$$2b^2 q_c = 4b^2(c_1 + \alpha c_1 + \beta c_2) + (q_c - c_1)b^2 - c_1 b^2 \qquad (A.13)$$

then
$$N_m = (2 + \pi - 4\beta) + 4\beta(c_2/c_1)$$

or
$$N_m = 4\beta n + 5.14 \qquad (A.14)$$

For the case d = b (A.12) and (A.14) yield the same value.

3B.2 Brown's Limit Plasticity Solutions

Brown (1967) presented an analysis of the modified-Prandtl type similar to the Tcheng-Prandtl solution described above. However, whereas Tcheng considered only boundary forces on the failure planes, Brown took into account the shear mobilized within the failure zones as well. By application of the theorems of limit plasticity, he was able to arrive at theoretical lower bound and upper bound solutions, thus bracketing a "true" solution.

a. Lower Bound Solution to the Two-Layer Clay Bearing Capacity Problem (Brown, 1967). By definition a lower bound solution (see Drucker and Prager, 1952) requires that a safe, "statically admissible" stress field be found. Such a stress field is obtained when the distributed stresses are in equilibrium with the applied loads and nowhere in the interior do they exceed the yield limit (Schofield and Wroth, 1968).

Brown's lower bound solution assumes the failure conditions as shown in Figure (A-7) below, and considers the internal shears which occur within the radial zone okl.

Although more sophisticated mathematical techniques* are available for obtaining plasticity solutions, one can adequately describe the stress changes through the shear zones by Kötter's (1903) equations.

Considering a weightless soil system, for $\phi = 0$ conditions, Kötter's equation becomes

$$\frac{\partial \sigma}{\partial s} \pm 2c \frac{\partial \theta}{\partial s} = 0$$

where

σ = average normal stress = $(\sigma_1 + \sigma_3)/2$.

c = an undrained shear strength, momentarily unspecified

θ,s, dimensions as noted in Figure (A-7c).

so,

$$\frac{\partial}{\partial s} \left(\frac{\sigma_1 + \sigma_3}{2}\right) \pm 2c \frac{\partial \theta}{\partial s} = 0 \qquad \text{(A.15)}$$

In the passive Rankine zone, pko, the slip lines are straight;

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*Specifically, those of Sokolovskii, V. V. (1965), *Statics of Granular Media,* Pergamon, London.

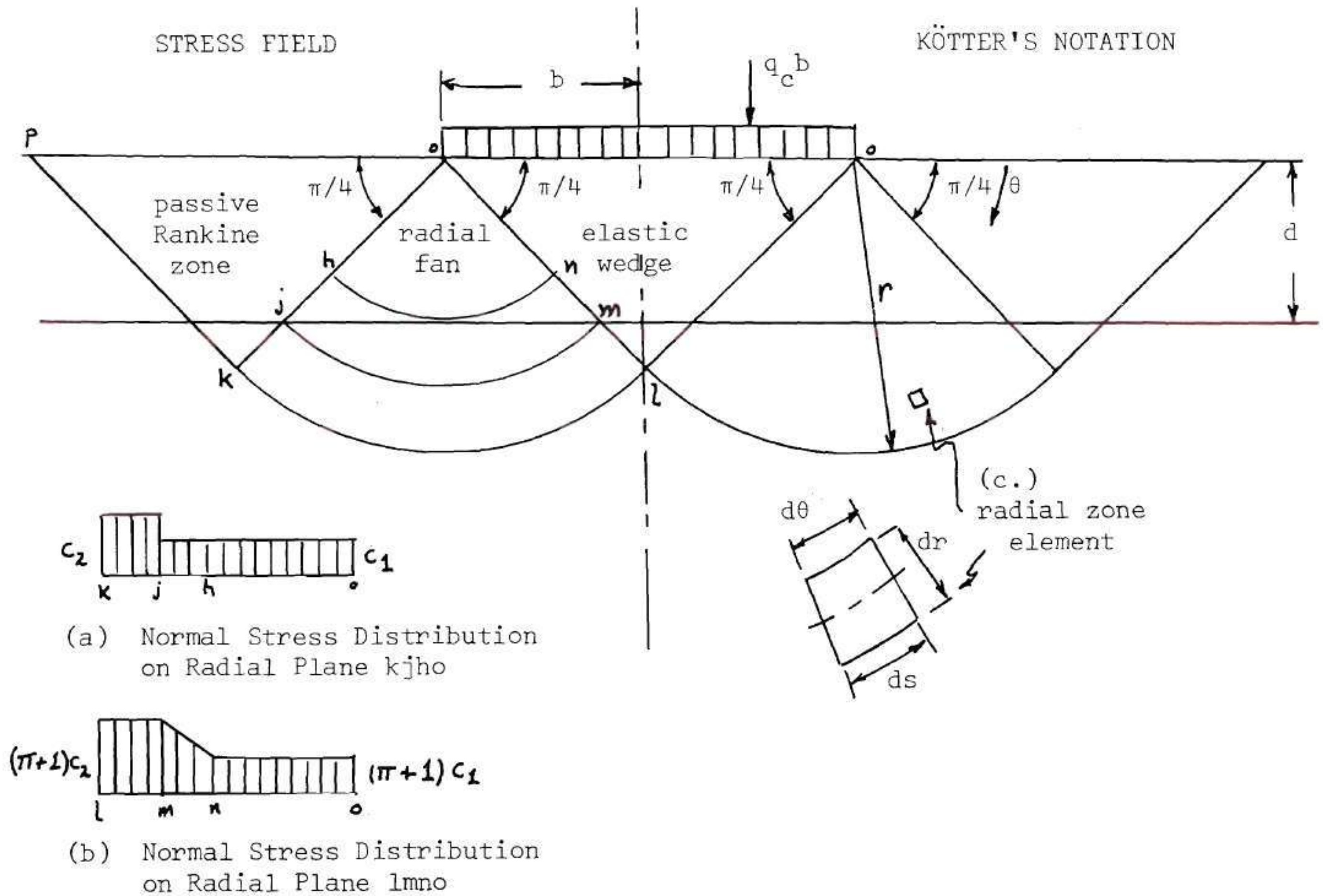


Figure A-7. Brown's Lower Bound Solution for the Bearing Capacity of a Two-Layer Clay hence,

$$\frac{\partial\theta}{\partial s} = 0$$

so by Equation (A.15),

$$\frac{\partial}{\partial s} \left(\frac{\sigma_1 + \sigma_3}{2}\right) = 0,$$

i.e., the average normal stress remains constant. Now in the adjacent radial shear zone okl, the slip surfaces are circular arcs, so that $r = s/\theta$ and

$$\frac{\partial\theta}{\partial s} = \frac{1}{r} = \text{Constant}.$$

Substituting this into Equation (A-15) yields

$$\frac{1}{r} \cdot \frac{\partial}{\partial\theta} \left(\frac{\sigma_1 + \sigma_3}{2}\right) - \frac{2c}{r} = 0,$$

or,

$$\frac{\partial}{\partial\theta} \left(\frac{\sigma_1 + \sigma_3}{2}\right) = 2c \tag{A.16}$$

When integrated, (A.16) yields

$$\frac{\sigma_1 + \sigma_3}{2} = 2c\theta + K$$

by noting the boundary condition $\theta = 0$; $\sigma_1 = 2c$, $\sigma_3 = 0$, evaluation of the above equation gives integration constant $K = c$ so that

$$\frac{\sigma_1 + \sigma_3}{2} = 2c\theta + c \tag{A.17}$$

for $\theta = 0$ (plane op) one obtains

$$\frac{\sigma_1 + \sigma_3}{2} = c \tag{A.18}$$

Since the average normal stress has previously been shown to be constant within Rankine zone pko, Equation (A.18) also applies to plane kjho. However, the layered soil system possesses no discrete "c" value; Brown assumed a variation as shown in Figure (A-7a).

In the radial zone ($\pi/4 \leq \theta \leq \frac{3\pi}{4}$), Equation (A.17) applies, so that for plane lmno

$$\left(\frac{\sigma_1 + \sigma_3}{2}\right) = c + 2c\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) = c(1 + \pi) \tag{A.19}$$

Again Brown made some assumptions regarding the "c" value of Equation (A.19); i.e., strength c_1 governs in zone ohn, c_2 governs in zone jklm, and hjmn is a "mixed" shear zone, assumed to vary linearly, as shown in Figure (A-7b). By summing vertical forces on the elastic wedge, held in equilibrium by the stress distribution of Figure (A-7b) and load $q_c b$, one obtains the lower bound solution:

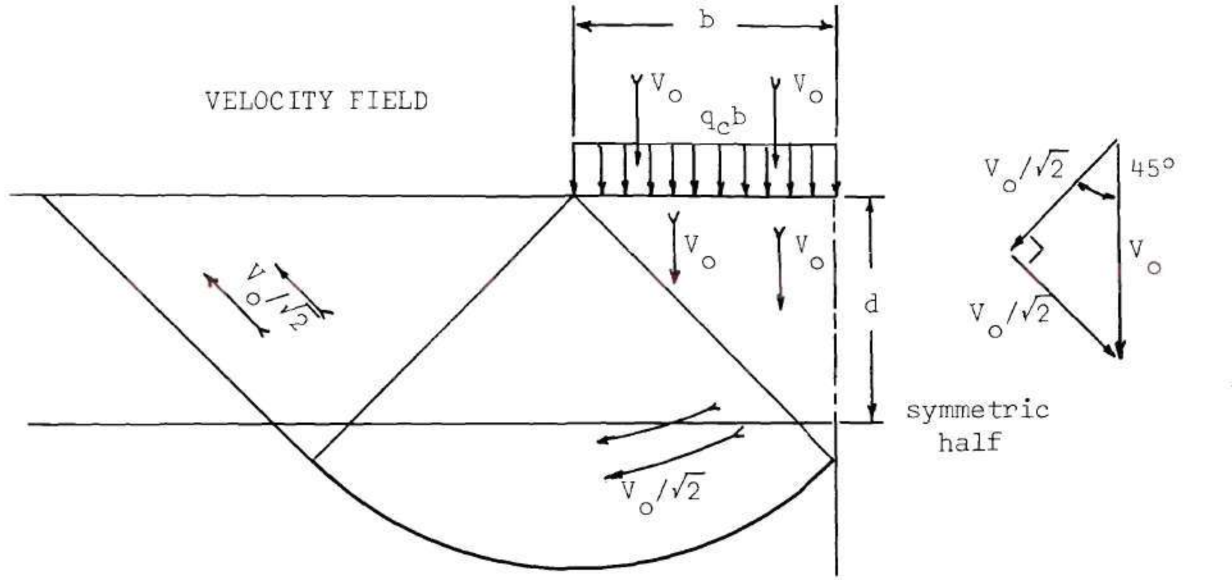
$$N_m = q_c/c_1 = 3.22\left(\frac{d}{b}\right) + (n+1)\ (5.14 - 3.22\ d/b) \qquad \text{(A.20)}$$

<u>b. Upper Bound Solution to the Two-Layer Clay Bearing Capacity Problem (Brown, 1967).</u> This approach involves finding a kinematically admissible velocity field, considered as plastic only, where the rate at which the external forces (surface tractions) do work on the body equals or exceeds the rate of internal dissipation (i.e., either failure impends or has already occurred) (Drucker and Prager, 1952).

The velocity field for a material which obeys the Mohr-Coulomb failure theory has been worked out by Shield (1953). In this 1953 paper he presents a punch-indentation problem which Brown has modified to the layered soil bearing capacity situation. The Prandtl geometry (Figure A-8) is again used in this solution. The plasticity theorems are valid only, where frictional surface tractions are present, if there is no slip (Drucker and Prager, 1951). Brown's analysis involves only one surface traction, $q_c b$ (symmetrical half taken in analysis). The work that this surface traction does is qbV_o. During failure this work is dissipated in two ways (Liam Finn, 1967): in the internal distortion of the radial zone, olk, and along the discontinuity olkp.

So that,

$$qbV_o \geq \int_o^A c\ \nu\ d\ A + \int_o^S c\ \Delta\ V_o\ dS \qquad \text{(A.21)}$$



NOTE: Fishtailed arrows denote velocities.

NOTE: Shield (1953) shows for ϕ = 0 all velocities are parallel to the slip lines.

Figure A-8. Brown's Upper Bound Solution for the Bearing Capacity of a Two-Layer Clay where

ν = maximum rate of shear strength,

A = area of radial zone,

S = length along surface discontinuity,

the velocity of the surface traction is taken as V_o (see Note, Figure A-8) then the velocity along ol is $V_o/\sqrt{2}$; there is no change in velocity along lk and kp thus the velocity everywhere along the discontinuity is $V_o/\sqrt{2}$.

So that:

$$\int_o^s c\Delta V_o ds = 2c_1 V_o d + c_2 V_o (b-d) + \left(\frac{\pi R}{2}\right) c_2 V_o$$

Since $R = b\sqrt{2}$

$$\int_o^S c\Delta V_o ds = 2c_1 V_o d + c_2 V_o \left(b-d + \frac{\pi b}{2}\right) \tag{A.22}$$

now

$$\int_o^A c\nu dA = 1.57\ c_2 V_o b - 1.29(c_2 - c_1) V_o d \tag{A.23}$$

These are solved to give, from Equation (A.21)

$$N_m = \frac{q}{c_t} \geq 3.29\ \frac{d}{b} + (n + 1)(5.14 - 3.29\ \frac{d}{b}) \tag{A.24}$$

By plastic limit analysis then, Brown has bracketed the true solution with Equations (A.20) and (A.24). However, he gives no indication where the true solution lies within this range; he takes the average of (A.20) and (A.24)

$$N_m = 3.25\,\frac{d}{b} + (n + 1)(5.14 - 3.25\ d/b) \qquad \text{(A.25)}$$

as the solution.

4. Meyerhof and Chaplin (1953) Solution

This method is treated separately, since, unlike the other solutions, it constitutes a new bearing capacity theory--one developed expressly for the layered soil problem. It is specifically for the case of a thin ($d/b \leq \frac{1}{2}$) layer of purely cohesive soil resting on a rough, rigid base.

Meyerhof and Chaplin extended an earlier Prandtl (1923) theory for the compression of plastic blocks between perfectly rough, rigid plates, applying it to the cases shown in Figure (A-9).

The case of interest in this thesis is that of a strip footing. As the ultimate pressure is reached, the material beneath a wide footing is displaced outward and upward as follows (Figure A-9a):

(1) central zones ABC are elastic wedges which act as a part of the footing and base;

(2) cycloidal shear zone ACD, merging into mixed shear zone EDG;

(3) radial zone FEGH, again merging into a mixed shear zone HKI.

With a narrow footing, the various zones coalesce as shown in Figure (A-9b).

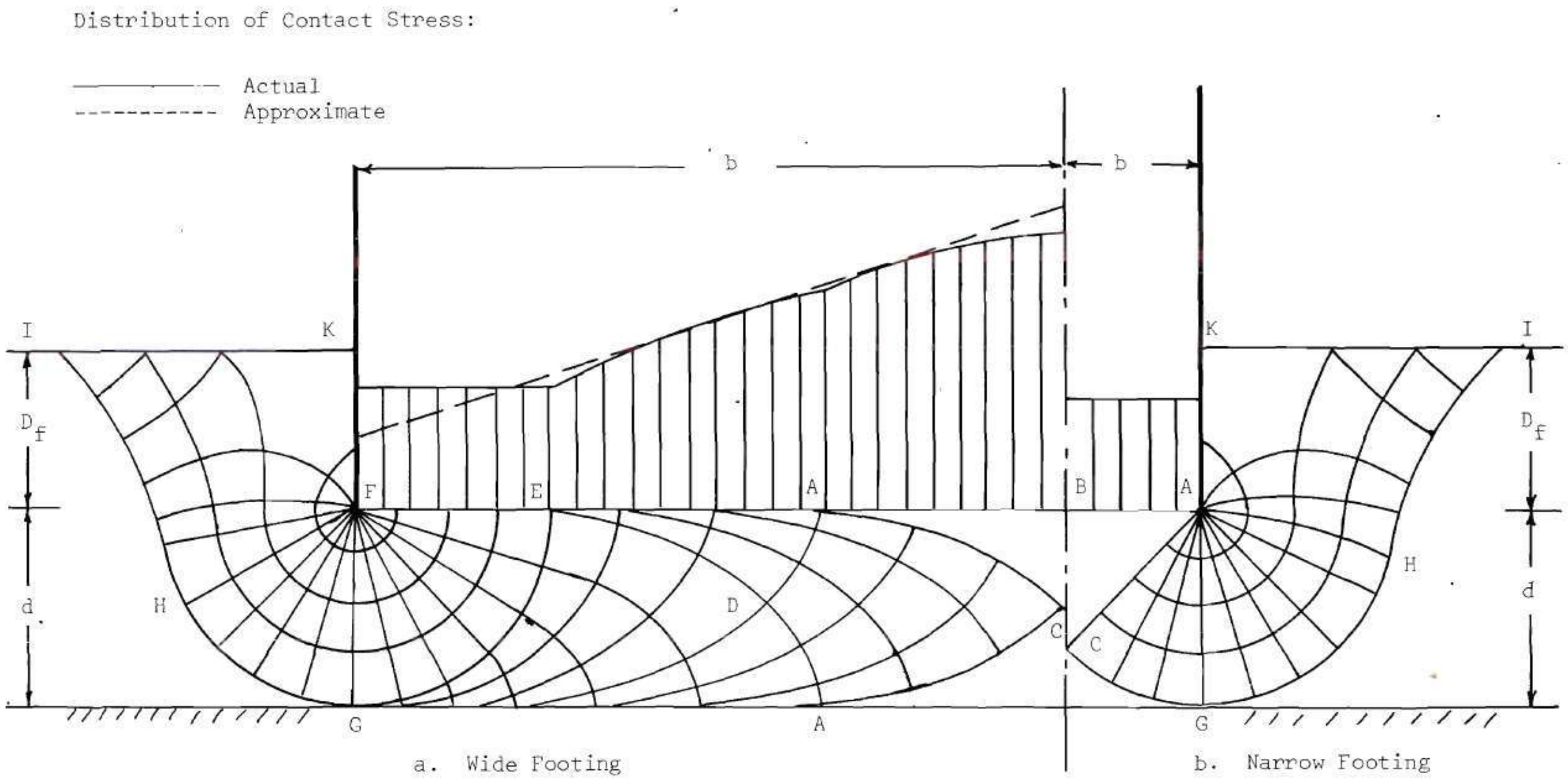


Figure A-9. Meyerhof and Chaplin Theory: Plastic Zones and Contact Pressure for a Perfectly Rough Footing on a Layer with a Perfectly Rough Base

The actual contact stress along the rough footing base theoretically varies with each of the zones of shear described above, however, Meyerhof and Chaplin note computational proof that the approximate (see Figure A-9) distribution, in which the contact pressure increases uniformly toward the center, gives an almost identical yield pressure as the more rigorous solution.*

For the case of $D_f = 0$, the radial shear zone merges into the Rankine passive zone of classical bearing capacity theory, i.e., the mixed shear HKI zone does not develop.

The ultimate bearing pressure is the sum of the yield pressure of the soil block under the footing (p_c) and the Rankine passive pressure in the soil outside the footing width (p_p), i.e.,

$$q_c = p_c + p_p \qquad \text{(A.26)}$$

Assuming the linear contact stress variation of Figure (A-9a), and that the cycloidal zone extends throughout the block, the average yield pressure is then

$$p_c = c(b/h + \pi/2)$$

and from the bearing capacity equation, for the Rankine wedge outside the footing:

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*Meyerhof and Chaplin cite Sokolovskii, V. V. (1946), *Theory of Plasticity*, Acad. Sci. U.S.S.R., Moscow.

$$p_p = (1 + \pi/2)c$$

by Equation (A-26)

$$q_c = (\pi + 1 + b/d)c$$

or

$$N_m = (\pi + 1 + b/d) \tag{A.27}$$

for $d/b \leq \frac{1}{2}$. Brown's (1967) tests later confirmed this as the limiting case, and he suggested

$$N_m = (\pi + 1 + 1.1\ b/d)$$

as a semiempirical equation to cover the remaining range for soft over stiff layer configuration.

5. Methods Based on Observations and Data from Model Studies

Two such modifications have been presented:

(a) Tcheng's (1956) Prandtl punching solution.

(b) Brown's (1967) aborted shear punching solution.

5a. Tcheng's (1956) Prandtl Punching Solution

Tcheng's model footing tests indicated that the Prandtl geometry of Figures A-5 and A-6 was not a good approximation to the observed failure surfaces. His tests for stiff over soft soil were carried out using grease of various consistencies (see Tcheng, 1957) as a lower layer. The soil system has a relatively high negative n value. He observed vertical punching through the top layer with the Prandtl configuration forming at the interface with the soft layer (Figure A-10)

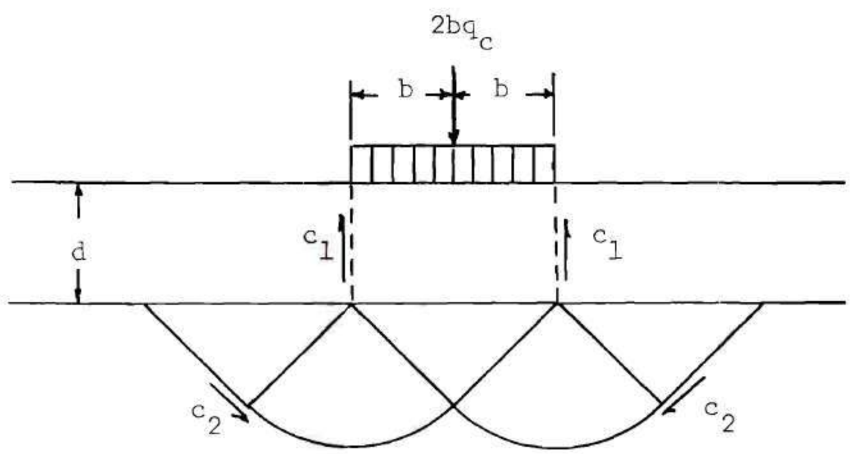

Figure A-10. Tcheng's Prandtl-Punching Solution

In this case,

$$q_c = 5.14\ c_2 + \frac{c_1 d}{b}\ ,$$

or,

$$N_m = d/b + 5.14\ (n + 1) \qquad (A.28)$$

5b. Brown's (1967) Aborted Shear Punching Solution

Brown (1967) performed a great number of model footing tests with the stiff over soft soil configuration, and found definite evidence of "punching" into the top layer. Brown's tests were conducted over a complete range of -n values, and he did not observe the *vertical* punching noted by Tcheng. Brown suggested that the soft underlying soil reaches bearing capacity before such "pure" punching occurs. What Brown observed is perhaps best termed as "aborted" shear punching; his semi-empirical equation uses 75 per cent of the simple vertical punching value (see Equation (A.28)) so that:

$$N_m = .75\ (d/b) + 5.14\ (n + 1) \tag{A.29}$$

APPENDIX B

UNCONFINED COMPRESSION TEST DATA,

REPRESENTATIVE STRESS-STRAIN CURVES

Table B-1. Summary: Strength Data* of Footing Tests

Test	d/b	c_1(psf)		c_2(psf)		Ratio Vane-UC	
		Vane	UC	Vane	UC	Top Layer	Bottom Layer
2HS-12	Stiff Soil	80	56	None	None	1.42	None
2HS-16	Soft Soil	40.5	43	None	None	0.95	None
2LS-1	0.5	80	78	30	28.5	1.02	1.03
2LS-2	0.5	80	78	30	28.5	1.02	1.03
2LS-5	0.5	80	-	36	35.2	-	1.02
2LS-6	0.5	80	-	36	35.2	-	1.02
2LL-21	0.5	58	46.5	44	23	1.22	1.90
2LB-22	0.5	93	77	53	44	1.22	1.23
2LS-3	1.0	80	-	48	36	-	1.33
2LS-4	1.0	80	-	48	36	-	1.33
2LS-9	1.0	80	-	54	36.8		1.28
2LS-17	2.0	55	55	40	38	1.00	1.15
2LB-14	2.0	74	-	53.3	-	-	-
2LS-7	3.0	80	49.7	73	46.7	1.62	1.56
2LS-8	3.0	84	63	70	44	1.33	1.59
2LB-11	3.0	91	-	64	-	-	-
2LS-10	0.6	26	25.3	40	40.25	1.01	0.99
2LS-13	0.6	24.5	25.9	-	56.2	0.95	-
2LS-14	0.6	32.5	26.0	43.0	55.1	1.25	0.77
2LB-18	0.6	-	33.8	-	54	-	-
2LL-20	0.6	50.5	47.6	-	-	-	1.03
2LS-15	0.8	42	35	69	42	1.2	1.6
2LL-23	0.8	40.5	48.8	48	58	84	-83

*See GLOSSARY OF SYMBOLS for notation. Each vane valve is the average of eight tests; each UC value is the average of four tests.

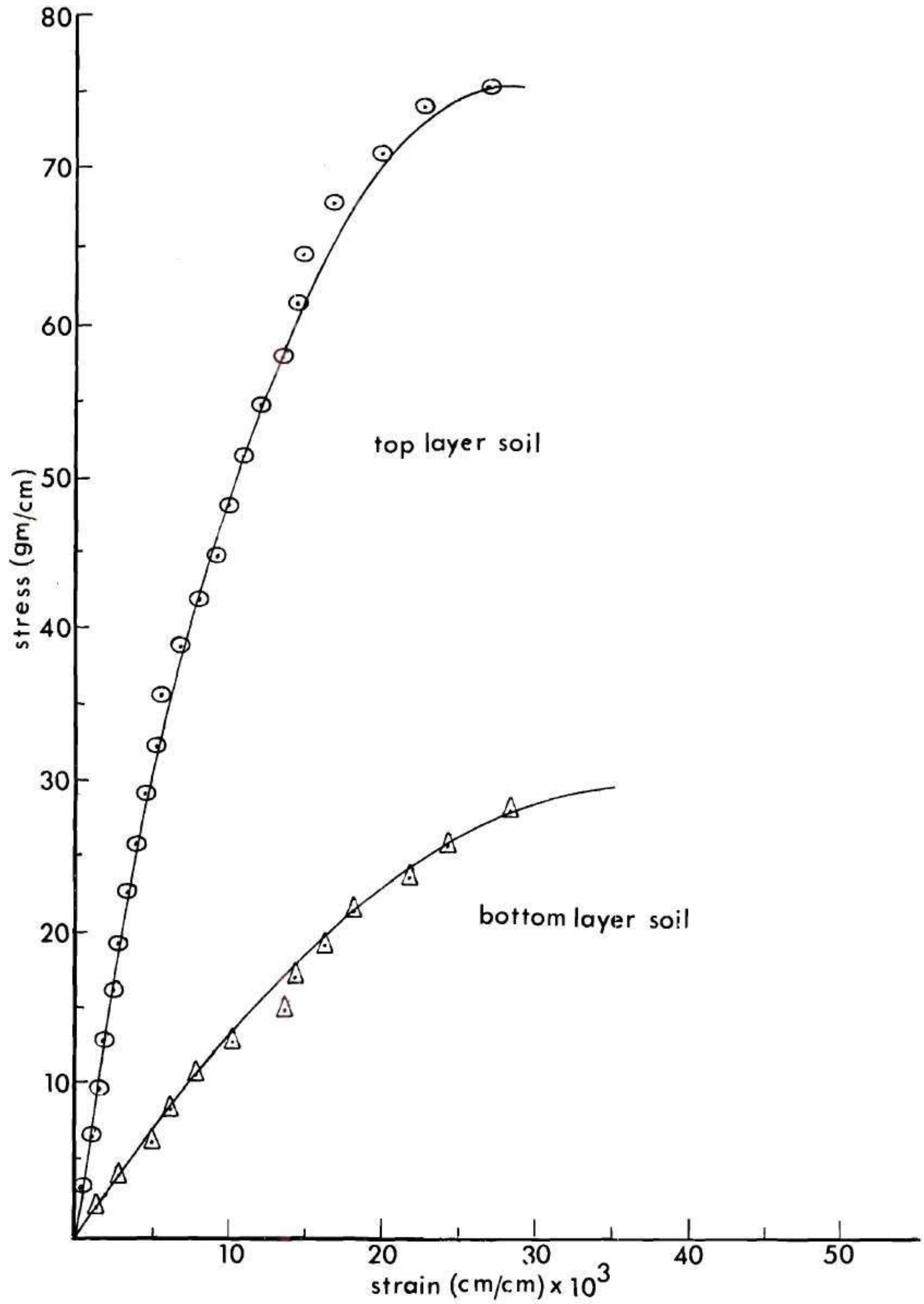

Figure B-2. Representative Stress-Strain Curves, Tests 2LS-1 and 2LS-2

REFERENCES CITED

Bjerrum, L. and A. Överland (1957), "Foundation Failure of an Oil Storage Tank in Fredrikstad, Norway," *Proceedings,* Fourth International Conference on Soil Mechanics and Foundation Engineering, London, Vol. 1, pp. 287-290.

Brown, John D. (1967), "The Ultimate Bearing Capacity of Layered Clay Foundations," Ph.D. Dissertation, Nova Scotia Tech. Coll. at Halifax.

Button, S. J. (1953), "The Bearing Capacity of Footings on a Two-Layer Cohesive Subsoil," *Proc.*, Third Inter. Conf. on Soil Mech. and Found. Eng., Zurich, Vol. 1, pp. 332-335.

Drucker, D. C. and W. Prager (1952), "Soil Mechanics and Plastic Analysis or Limit Design," *Quar. Appl. Math.,* Vol. X, No. 2, pp. 157-165.

Eden, W. J. and M. Bozozuk (1962), "Foundation Failure of a Silo on Varved Clay," *Engineering Journal,* pp. 54-57.

Fenske, C. W. (1957), "Deep Vane Tests in Gulf of Mexico," Symposium on Vane Shear Testing of Soils, ASTM *Spec. Tech. Publ.* 193, pp. 16-25.

Finn, W. D. Liam (1967), "Application of Limit Plasticity in Soil Mechanics," *Proc.* ASCE, Vol. 93, SM-5, pp. 101-119.

Flaate, Kaare S. (1966), "Factors Influencing the Results of Vane Tests," *Canadian Geotechnical Journal,* Vol. III, No. 1, pp. 18-31.

Golder, H. Q. (1941), "The Ultimate Bearing Pressure of Rectangular Footings," *Jour. Inst. of Civ. Eng.,* Vol. 17, p. 161.

Goughnour, R. D. and J. R. Sallberg (1964), "Evaluation of the Laboratory Vane Shear Test," *Highway Res. Bd. Record* 48, pp. 19-33.

Gray, H. (1955), "Field Vane Shear Tests of Sensitive Cohesive Soils," *Proc.* ASCE, Vol. 81, No. 755.

Haythornthwaite, R. M. (1961), "Method of Plasticity in Land Locomotion Studies," *Proc.* First Inter. Conf. Mech. Soil-Vehicle Systems, Turin, pp. 3019.

Koizumi, Y. (1965), "Discussion," in Shallow Found. Div., *Proc.,* Sixth Inter. Conf. on Soil Mech. and Found. Eng., Montreal, Vol. 3, pp. 413-415.

Kötter, F. (1903), Berlin Akad. der Wiss. (cited by: George F. Sowers (1968), Unpublished class notes--Theoretical Soil Mechanics, School of Civil Engineering, Grad. Div., Ga. Inst. of Tech., Atlanta).

Lo, K. Y. and V. Milligan (1967), "Shear Strength Properties of Two Stratified Clays," *Proc.* ASCE, Vol. 93, SM-1, pp. 1-15.

Marsal, R. J. (1957), "Unconfined Compression and Vane Shear Tests in Volcanic Lacustrine Clays," Conf. on Soils for Eng. Purposes, ASTM *Spec. Tech. Publ.* 232, pp. 229-241.

Meyerhof, G. G. (1950), "The Bearing Capacity of Sand," Ph.D. Diss., University of London.

Meyerhof, G. G. and John D. Brown (1967), Discussion of (cited) paper by Srinivasan and Siva Reddy (1967), *Proc.* ASCE, Vol. 93, SM-5, p. 361.

Meyerhof, George G. and T. K. Chaplin (1953), "The Compression and Bearing Capacity of Cohesive Layers," *Brit. Jour. of Appl. Physics,* Vol. 4, pp. 20-26.

Miller, E. T. (1957), "Vane Tests of Soils of Southern Piedmont," M.S.C.E. Thesis, Ga. Inst. of Tech., Atlanta.

Nixon, I. K. (1949), "Correspondence," (ϕ = 0 Analysis) *Geotechnique,* Vol. 1, Nos. 3 and 4, pp. 208 and 272-276.

Osterberg, J. O. (1957), "Introduction," Symposium on Vane Shear Testing of Soils, ASTM *Spec. Tech. Publ.* 193, pp. 1-7.

Prandtl, L. (1920), "Uber die Haerte Plasticher Koerper," Nachr. Kgl. Ges. Wiss. Goettingen, Math-Phys. Kl. (as cited in Alfreds R. Jumikis (1967), *Introduction to Soil Mechanics,* Van Nostrand Co., pp. 331-333).

Prandtl, L. (1923), Z. angew. Math. u. Mech., Vol. 3 (cited in Hoffman and Sachs (1956), *Theory of Plasticity,* McGraw-Hill, New York, pp. 129-132.

Ramanathan, B. (1967), Discussion of the (cited) paper by Srinivasan and Siva Reddy (1967), *Proc.* ASCE, Vol. 93, SM-6, p. 390.

Raymond, G. P. (1967), "The Bearing Capacity of Large Footings and Embankments on Clays," *Geotechnique,* Vol. 17, pp. 1-10.

Roberts, J. E. (1961), "Small Scale Footing Studies: A Review of the Literature," *Air Force Special Weapons Center,* Tech. Report 61-48.

Rocha, M. and J. Folque (1953), "Some Results of Settlement Observations in Actual Structures and in Models," Pub. No. 36, Laboratorio Nacional Engenhiria Civil, Lisbon.

Saurin, B. F. (1949), "Correspondence," (ϕ = 0 Analysis) *Geotechnique,* Vol. 1, No. 4, pp. 272-274.

Schofield, A. N. and C. P. Wroth (1968), *Critical State Soil Mechanics,* McGraw-Hill Co., New York.

Shield, R. T. (1953), "Mixed Boundary Value Problems in Soil Mech.," *Quar. Appl. Math.,* Vol. XI, No. 1, pp. 61-75.

Skempton, A. W. (1948), "The ϕ = 0 Analysis of Stability and Its Theoretical Basis," *Proc.* Second Inter. Conf. on Soil Mech. and Found. Eng., Rotterdam, Vol. 1, pp. 72-77.

Skempton, A. W. (1951), "The Bearing Capacity of Clays," *Proc.* Bldg. Res. Cong., London, pp. 180-189.

Sowers, G. F. (1963), "Shallow Foundations," Chap. Six in G. Leonards (ed.), *Foundation Engineering,* McGraw-Hill Co., New York.

Sowers, G. B. and G. F. Sowers (1961), *Introductory Soil Mechanics and Foundations,* MacMillan, New York.

Srinivasan, R. and A. Siva Reddy (1967), "Bearing Capacity of Footings on Layered Clays," *Proc.* ASCE, Vol. 93, SM-2, pp. 83-99.

Szechy, C. (1961), *Foundation Failures,* Concrete Publications, Ltd., London.

Taylor, Donald W. (1948), *Fundamentals of Soil Mechanics,* Art. 19-28, Wiley, New York.

Tcheng, Yuan (1956), "Pouvoir Portant dùn Solide Composé de Deux Couches Plastiques Différentes," Ph.D. Diss., Univ. of Paris (cited by Brown (1967), *op. cit.*).

Tcheng, Yuan (1957), "Fondations Superficielles en Milieu Stratifié," *Proc.* Fourth Inter. Conf. on Soil Mech. and Found. Eng., London, Vol. 1, pp. 449-452.

Terzaghi, Karl T. (1943), *Theoretical Soil Mechanics,* Arts. 45-46, Wiley, New York.

Terzaghi, K. and R. B. Peck (1968), *Soil Mechanics in Engineering Practice,* Art. 54, Wiley, New York.

Tchebortarioff, G. P. (1951), *Soil Mechanics, Foundations and Earth Structures,* Art. 9-9, McGraw-Hill, New York.

Yamaguchi, Hakuju (1963), "Practical Formula of Bearing Value for Two-Layered Ground," *Proc.* Second Asian Regional Conf. on Soil Mech., pp. 176-180.